Practice Problems

11.1 The Many Forms of Energy 
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1. A skater with a mass of 52.0 kg moving at 2.5 m/s glides to a stop over a distance of 24.0 m. How much work did the friction of the ice do to bring the skater to a stop? How much work would the skater have to do to speed up to 2.5 m/s again?

To bring the skater to a stop:

\[ W = KE_f - KE_i \]

\[ = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2 \]

\[ = \frac{1}{2} (52.0 \text{ kg})(0.00 \text{ m/s})^2 - \frac{1}{2} (52.0 \text{ kg})(2.5 \text{ m/s})^2 \]

\[ = -160 \text{ J} \]

To speed up again:

This is the reverse of the first question.

\[ W = KE_f - KE_i \]

\[ = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2 \]

\[ = \frac{1}{2} (52.0 \text{ kg})(2.5 \text{ m/s})^2 - \frac{1}{2} (52.0 \text{ kg})(0.00 \text{ m/s})^2 \]

\[ = +160 \text{ J} \]

2. An 875.0-kg compact car speeds up from 22.0 m/s to 44.0 m/s while passing another car. What are its initial and final energies, and how much work is done on the car to increase its speed?

The initial kinetic energy of the car is

\[ KE_i = \frac{1}{2} mv^2 = \frac{1}{2} (875.0 \text{ kg})(22.0 \text{ m/s})^2 \]

\[ = 2.12 \times 10^5 \text{ J} \]

The final kinetic energy is

\[ KE_f = \frac{1}{2} mv^2 = \frac{1}{2} (875.0 \text{ kg})(44.0 \text{ m/s})^2 \]

\[ = 8.47 \times 10^5 \text{ J} \]

The work done is

\[ KE_f - KE_i = 8.47 \times 10^5 \text{ J} - 2.12 \times 10^5 \text{ J} \]

\[ = 6.35 \times 10^5 \text{ J} \]

3. A comet with a mass of \(7.85 \times 10^{11}\) kg strikes Earth at a speed of 25.0 km/s. Find the kinetic energy of the comet in joules, and compare the work that is done by Earth in stopping the comet to the \(4.2 \times 10^{15}\) J of energy that was released by the largest nuclear weapon ever built.

\[ KE = \frac{1}{2} mv^2 \]

\[ = \frac{1}{2} (7.85 \times 10^{11} \text{ kg})(2.5 \times 10^4 \text{ m/s})^2 \]

\[ = 2.45 \times 10^{20} \text{ J} \]

\[ KE_{\text{comet}} = \frac{2.45 \times 10^{20} \text{ J}}{4.2 \times 10^{15} \text{ J}} = 5.8 \times 10^4 \]

\(5.8 \times 10^4\) bombs would be required to produce the same amount of energy used by Earth in stopping the comet.

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4. In Example Problem 1, what is the potential energy of the bowling ball relative to the rack when it is on the floor?

\[ PE = mgh \]

\[ = (7.30 \text{ kg})(9.80 \text{ m/s}^2)(-0.610 \text{ m}) \]

\[ = -43.6 \text{ J} \]

5. If you slowly lower a 20.0-kg bag of sand 1.20 m from the trunk of a car to the driveway, how much work do you do?

\[ W = Fd \]

\[ = mg(h_f - h_i) \]

\[ = (20.0 \text{ kg})(9.80 \text{ m/s}^2)(0.00 \text{ m} - 1.20 \text{ m}) \]

\[ = -2.35 \times 10^2 \text{ J} \]
6. A boy lifts a 2.2-kg book from his desk, which is 0.80 m high, to a bookshelf that is 2.10 m high. What is the potential energy of the book relative to the desk?

\[ PE = mg(h_f - h_i) \]
\[ = (2.2 \text{ kg})(9.80 \text{ m/s}^2)(2.10 \text{ m} - 0.80 \text{ m}) \]
\[ = 28 \text{ J} \]

7. If a 1.8-kg brick falls to the ground from a chimney that is 6.7 m high, what is the change in its potential energy?

Choose the ground as the reference level.

\[ \Delta PE = mg(h_f - h_i) \]
\[ = (1.8 \text{ kg})(9.80 \text{ m/s}^2)(0.0 \text{ m} - 6.7 \text{ m}) \]
\[ = -1.2 \times 10^2 \text{ J} \]

8. A warehouse worker picks up a 10.1-kg box from the floor and sets it on a long, 1.1-m-high table. He slides the box 5.0 m along the table and then lowers it back to the floor. What were the changes in the energy of the box, and how did the total energy of the box change? (Ignore friction.)

To lift the box to the table:

\[ W = Fd \]
\[ = mg(h_f - h_i) \]
\[ = \Delta PE \]
\[ = (10.1 \text{ kg})(9.80 \text{ m/s}^2)(1.1 \text{ m} - 0.0 \text{ m}) \]
\[ = 1.1 \times 10^2 \text{ J} \]

To slide the box across the table, \( W = 0.0 \) because the height did not change and we ignored friction.

To lower the box to the floor:

\[ W = Fd \]
\[ = mg(h_f - h_i) \]
\[ = \Delta PE \]
\[ = (10.1 \text{ kg})(9.80 \text{ m/s}^2)(0.0 \text{ m} - 1.1 \text{ m}) \]
\[ = -1.1 \times 10^2 \text{ J} \]

The sum of the three energy changes is 
\[ 1.1 \times 10^2 \text{ J} + 0.0 \text{ J} + (-1.1 \times 10^2 \text{ J}) = 0.0 \text{ J} \]

Section Review

11.1 The Many Forms of Energy pages 285–292

9. Elastic Potential Energy You get a spring-loaded toy pistol ready to fire by compressing the spring. The elastic potential energy of the spring pushes the rubber dart out of the pistol. You use the toy pistol to shoot the dart straight up. Draw bar graphs that describe the forms of energy present in the following instances.

a. The dart is pushed into the gun barrel, thereby compressing the spring.

```
Spring  | Dart  | Dart
elastic | gravitational | kinetic
potential energy | potential energy | energy

There should be three bars: one for the spring’s potential energy, one for gravitational potential energy, and one for kinetic energy. The spring’s potential energy is at the maximum level, and the other two are zero.
```

b. The spring expands and the dart leaves the gun barrel after the trigger is pulled.

```
Spring  | Dart  | Dart
elastic | gravitational | kinetic
potential energy | potential energy | energy

The kinetic energy is at the maximum level, and the other two are zero.
```
c. The dart reaches the top of its flight.

The gravitational potential energy is at the maximum level, and the other two are zero.

10. **Potential Energy**  
A 25.0-kg shell is shot from a cannon at Earth’s surface. The reference level is Earth’s surface. What is the gravitational potential energy of the system when the shell is at 425 m? What is the change in potential energy when the shell falls to a height of 225 m?

a. \[
PE = mgh \\
= (25.0 \text{ kg})(9.80 \text{ m/s}^2)(425 \text{ m}) \\
= 1.04 \times 10^5 \text{ J}
\]

b. \[
PE = mgh \\
= (25.0 \text{ kg})(9.80 \text{ m/s}^2)(225 \text{ m}) \\
= 5.51 \times 10^4 \text{ J}
\]

The change in energy is

\[
(1.04 \times 10^5 \text{ J}) - (5.51 \times 10^4 \text{ J}) = 4.89 \times 10^4 \text{ J}
\]

11. **Rotational Kinetic Energy**  
Suppose some children push a merry-go-round so that it turns twice as fast as it did before they pushed it. What are the relative changes in angular momentum and rotational kinetic energy?

The angular momentum is doubled because it is proportional to the angular velocity. The rotational kinetic energy is quadrupled because it is proportional to the square of the angular velocity. The children did work in rotating the merry-go-round.

12. **Work-Energy Theorem**  
How can you apply the work-energy theorem to lifting a bowling ball from a storage rack to your shoulder?

The bowling ball has zero kinetic energy when it is resting on the rack or when it is held near your shoulder. Therefore, the total work done on the ball by you and by gravity must equal zero.

13. **Potential Energy**  
A 90.0-kg rock climber first climbs 45.0 m up to the top of a quarry, then descends 85.0 m from the top to the bottom of the quarry. If the initial height is the reference level, find the potential energy of the system (the climber and Earth) at the top and at the bottom. Draw bar graphs for both situations.

\[
PE = mgh \\
= (90.0 \text{ kg})(9.80 \text{ m/s}^2)(45.0 \text{ m}) \\
= 3.97 \times 10^4 \text{ J}
\]

At the bottom,

\[
PE = (90.0 \text{ kg})(9.80 \text{ m/s}^2) (+45.0 \text{ m} - 85.0 \text{ m}) \\
= -3.53 \times 10^4 \text{ J}
\]

14. **Critical Thinking**  
Karl uses an air hose to exert a constant horizontal force on a puck, which is on a frictionless air table. He keeps the hose aimed at the puck, thereby creating a constant force as the puck moves a fixed distance.

a. Explain what happens in terms of work and energy. Draw bar graphs.

\[
KE_{initial} + W = KE_{final}
\]

Karl exerted a constant force \( F \) over a distance \( d \) and did an amount of work \( W = Fd \) on the puck. This work changed the kinetic energy of the puck.
Chapter 11 continued

$$W = (KE_i - KE_f)$$

$$= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$= \frac{1}{2} m v_f^2$$

b. Suppose Karl uses a different puck with half the mass of the first one. All other conditions remain the same. How will the kinetic energy and work differ from those in the first situation?

If the puck has half the mass, it still receives the same amount of work and has the same change in kinetic energy. However, the smaller mass will move faster by a factor of 1.414.

c. Explain what happened in parts a and b in terms of impulse and momentum.

The two pucks do not have the same final momentum.

Momentum of the first puck:

$$p_1 = m_1 v_1$$

Momentum of the second puck:

$$p_2 = m_2 v_2$$

$$= \left(\frac{1}{2} m_1\right)(1.414 v_1)$$

$$= 0.707 p_1$$

Thus, the second puck has less momentum than the first puck does. Because the change in momentum is equal to the impulse provided by the air hose, the second puck receives a smaller impulse.

Practice Problems

11.2 Conservation of Energy pages 293–301

15. A bike rider approaches a hill at a speed of 8.5 m/s. The combined mass of the bike and the rider is 85.0 kg. Choose a suitable system. Find the initial kinetic energy of the system. The rider coasts up the hill. Assuming there is no friction, at what height will the bike come to rest?

The system is the bike + rider + Earth. There are no external forces, so total energy is conserved.

$$KE = \frac{1}{2} m v^2$$

$$= \frac{1}{2} (85.0 \text{ kg})(8.5 \text{ m/s})^2$$

$$= 3.1 \times 10^3 \text{ J}$$

$$KE_i + PE_i = KE_f + PE_f$$

$$\frac{1}{2} m v^2 + 0 = 0 + mgh$$

$$h = \frac{v^2}{2g} = \frac{(8.5 \text{ m/s})^2}{(2)(9.80 \text{ m/s}^2)}$$

$$= 3.7 \text{ m}$$

16. Suppose that the bike rider in problem 15 pedaled up the hill and never came to a stop. In what system is energy conserved? From what form of energy did the bike gain mechanical energy?

The system of Earth, bike, and rider remains the same, but now the energy involved is not mechanical energy alone. The rider must be considered as having stored energy, some of which is converted to mechanical energy.

Energy came from the chemical potential energy stored in the rider’s body.

17. A skier starts from rest at the top of a 45.0-m-high hill, skis down a 30° incline into a valley, and continues up a 40.0-m-high hill. The heights of both hills are measured from the valley floor. Assume that you can neglect friction and the effect of the ski poles. How fast is the skier moving at the bottom of the valley? What is the skier’s speed at the top of the next hill? Do the angles of the hills affect your answers?

Bottom of valley:

$$KE_i + PE_i = KE_f + PE_f$$

$$0 + mgh = \frac{1}{2} m v^2 + 0$$

$$v^2 = 2gh$$

$$v = \sqrt{2gh}$$

$$= \sqrt{(2)(9.80 \text{ m/s}^2)(45.0 \text{ m})}$$
Chapter 11 continued

Top of next hill:
\[ KE_i + PE_i = KE_f + PE_f \]
\[ 0 + mgh_i = \frac{1}{2} mv^2 + mgh_f \]
\[ v^2 = 2g(h_i - h_f) \]
\[ = \sqrt{2g(h_i - h_f)} \]
\[ = \sqrt{(2)(9.80 \text{ m/s}^2)(45.0 \text{ m} - 40.0 \text{ m})} \]
\[ = 9.90 \text{ m/s} \]

No, the angles do not have any impact.

18. In a belly-flop diving contest, the winner is the diver who makes the biggest splash upon hitting the water. The size of the splash depends not only on the diver’s style, but also on the amount of kinetic energy that the diver has. Consider a contest in which each diver jumps from a 3.00-m platform. One diver has a mass of 136 kg and simply steps off the platform. Another diver has a mass of 102 kg and leaps upward from the platform. How high would the second diver have to leap to make a competitive splash?

Using the water as a reference level, the kinetic energy on entry is equal to the potential energy of the diver at the top of his flight. The large diver has \( PE = mgh = (136 \text{ kg})(9.80 \text{ m/s}^2)(3.00 \text{ m}) = 4.00 \times 10^3 \text{ J} \)

To equal this, the smaller diver would have to jump to
\[ h = \frac{4.00 \times 10^3 \text{ J}}{(102 \text{ kg})(9.80 \text{ m/s}^2)} = 4.00 \text{ m} \]

Thus, the smaller diver would have to leap 1.00 m above the platform.

19. An 8.00-g bullet is fired horizontally into a 9.00-kg block of wood on an air table and is embedded in it. After the collision, the block and bullet slide along the frictionless surface together with a speed of 10.0 cm/s. What was the initial speed of the bullet?

Conservation of momentum:
\[ mv = (m + M)V, \text{ or} \]
\[ v = \frac{(m + M)V}{m} \]
\[ = \frac{(0.00800 \text{ kg} + 9.00 \text{ kg})(0.100 \text{ m/s})}{0.00800 \text{ kg}} \]
\[ = 1.13 \times 10^2 \text{ m/s} \]

20. A 0.73-kg magnetic target is suspended on a string. A 0.025-kg magnetic dart, shot horizontally, strikes the target head-on. The dart and the target together, acting like a pendulum, swing 12.0 cm above the initial level before instantaneously coming to rest.

a. Sketch the situation and choose a system.

The system includes the suspended target and the dart.

b. Decide what is conserved in each part and explain your decision.

Only momentum is conserved in the inelastic dart-target collision, so
\[ mv_i + MV_i = (m + M)V_f \]
where \( V_f = 0 \) since the target is initially at rest and \( V_i \) is the common velocity just after impact. As the dart-target combination swings upward, energy is conserved, so
\[ \Delta PE = \Delta KE \text{ or, at the top of the swing,} \]
\[ (m + M)gh_f = \frac{1}{2} (m + M)(V_f)^2 \]
c. What was the initial velocity of the dart?

Solve for $V_f$.

$$V_f = \sqrt{2gh_f}$$

Substitute $v_f$ into the momentum equation and solve for $v_i$.

$$v_i = \left(\frac{m + M}{m}\right)\sqrt{2gh_f}$$

$$= \left(\frac{0.025 \text{ kg} + 0.73 \text{ kg}}{0.025 \text{ kg}}\right)\left(\sqrt{(2)(9.80 \text{ m/s}^2)(0.120 \text{ m})}\right)$$

$$= 46 \text{ m/s}$$

21. A 91.0-kg hockey player is skating on ice at 5.50 m/s. Another hockey player of equal mass, moving at 8.1 m/s in the same direction, hits him from behind. They slide off together.

a. What are the total energy and momentum in the system before the collision?

$$KE_i = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$= \frac{1}{2} (91.0 \text{ kg})(5.50 \text{ m/s})^2 + \frac{1}{2} (91.0 \text{ kg})(8.1 \text{ m/s})^2$$

$$= 4.4 \times 10^3 \text{ J}$$

$$p_i = m_1 v_1 + m_2 v_2$$

$$= (91.0 \text{ kg})(5.5 \text{ m/s}) + (91.0 \text{ kg})(8.1 \text{ m/s})$$

$$= 1.2 \times 10^3 \text{ kg} \cdot \text{m/s}$$

b. What is the velocity of the two hockey players after the collision?

After the collision:

$$p_i = p_f$$

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$= \frac{(91.0 \text{ kg})(5.50 \text{ m/s}) + (91.0 \text{ kg})(8.1 \text{ m/s})}{91.0 \text{ kg} + 91.0 \text{ kg}}$$

$$= 6.8 \text{ m/s}$$

c. How much energy was lost in the collision?

The final kinetic energy is

$$KE_f = \frac{1}{2} (m_1 + m_2) v_f^2$$

$$= \frac{1}{2} (91.0 \text{ kg} + 91.0 \text{ kg})(6.8 \text{ m/s})^2$$

$$= 4.2 \times 10^3 \text{ J}$$

Thus, the energy lost in the collision is

$$KE_i - KE_f = 4.4 \times 10^3 \text{ J} - 4.2 \times 10^3 \text{ J}$$

$$= 2 \times 10^2 \text{ J}$$
Section Review

11.2 Conservation of Energy pages 293–301

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To simplify problems that take place over a short time, Earth is considered a closed system. It is not actually isolated, however, because it is acted upon by the gravitational forces from the planets, the Sun, and other stars. In addition, Earth is the recipient of continuous electromagnetic energy, primarily from the Sun.

23. Energy A child jumps on a trampoline. Draw bar graphs to show the forms of energy present in the following situations.

   a. The child is at the highest point.

   b. The child is at the lowest point.

24. Kinetic Energy Suppose a glob of chewing gum and a small, rubber ball collide head-on in midair and then rebound apart. Would you expect kinetic energy to be conserved? If not, what happens to the energy?

   Even though the rubber ball rebound with little energy loss, kinetic energy would not be conserved in this case because the glob of chewing gum probably was deformed in the collision.

25. Kinetic Energy In table tennis, a very light but hard ball is hit with a hard rubber or wooden paddle. In tennis, a much softer ball is hit with a racket. Why are the two sets of equipment designed in this way? Can you think of other ball-paddle pairs in sports? How are they designed?

   The balls and the paddle and racket are designed to match so that the maximum amount of kinetic energy is passed from the paddle or racket to the ball. A softer ball receives energy with less loss from a softer paddle or racket. Other combinations are a golf ball and club (both hard) and a baseball and bat (also both hard).

26. Potential Energy A rubber ball is dropped from a height of 8.0 m onto a hard concrete floor. It hits the floor and bounces repeatedly. Each time it hits the floor, it loses $\frac{1}{5}$ of its total energy. How many times will it bounce before it bounces back up to a height of only about 4 m?

   Since the rebound height is proportional to energy, each bounce will rebound to $\frac{4}{5}$ the height of the previous bounce.

   After one bounce: $h = \left(\frac{4}{5}\right)(8 \text{ m}) = 6.4 \text{ m}$

   After two bounces: $h = \left(\frac{4}{5}\right)(6.4 \text{ m}) = 5.12 \text{ m}$

   After three bounces: $h = \left(\frac{4}{5}\right)(5.12 \text{ m}) = 4.1 \text{ m}$
27. **Energy**  
As shown in Figure 11-15, a 36.0-kg child slides down a playground slide that is 2.5 m high. At the bottom of the slide, she is moving at 3.0 m/s. How much energy was lost as she slid down the slide?

![Figure 11-15](image)

\[ E_i = mgh \]
\[ = (36.0 \text{ kg})(9.80 \text{ m/s}^2)(2.5 \text{ m}) \]
\[ = 880 \text{ J} \]

\[ E_f = \frac{1}{2}mv^2 \]
\[ = \frac{1}{2}(36.0 \text{ kg})(3.0 \text{ m/s})^2 \]
\[ = 160 \text{ J} \]

Energy loss = 880 J – 160 J
\[ = 720 \text{ J} \]

28. **Critical Thinking**  
A ball drops 20 m. When it has fallen half the distance, or 10 m, half of its energy is potential and half is kinetic. When the ball has fallen for half the amount of time it takes to fall, will more, less, or exactly half of its energy be potential energy?

The ball falls more slowly during the beginning part of its drop. Therefore, in the first half of the time that it falls, it will not have traveled half of the distance that it will fall. Therefore, the ball will have more potential energy than kinetic energy.

### Chapter Assessment

#### Concept Mapping

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Complete the concept map using the following terms: gravitational potential energy, elastic potential energy, kinetic energy.

![Concept Map](image)

#### Mastering Concepts

**page 306**

Unless otherwise directed, assume that air resistance is negligible.

29. **Critical Thinking**  
A ball drops 20 m. When it has fallen half the distance, or 10 m, half of its energy is potential and half is kinetic. When the ball has fallen for half the amount of time it takes to fall, will more, less, or exactly half of its energy be potential energy?

The ball falls more slowly during the beginning part of its drop. Therefore, in the first half of the time that it falls, it will not have traveled half of the distance that it will fall. Therefore, the ball will have more potential energy than kinetic energy.

30. **Explain how work and a change in energy are related.** (11.1)

The work done on an object causes a change in the object’s energy. This is the work-energy theorem.

31. **What form of energy does a wound-up watch spring have? What form of energy does a functioning mechanical watch have?**  
When a watch runs down, what has happened to the energy? (11.1)

The wound-up watch spring has elastic potential energy. The functioning watch has elastic potential energy and rotational kinetic energy. The watch runs down when all of the energy has been converted to heat by friction in the gears and bearings.

32. **Explain how energy change and force are related.** (11.1)

A force exerted over a distance does work, which produces a change in energy.

33. **A ball is dropped from the top of a building. You choose the top of the building to be the reference level, while your friend chooses the bottom. Explain whether the**
energy calculated using these two reference levels is the same or different for the following situations. (11.1)

a. the ball's potential energy at any point
   The potential energies are different due to the different reference levels.

b. the change in the ball's potential energy as a result of the fall
   The changes in the potential energies as a result of the fall are equal because the change in $h$ is the same for both reference levels.

c. the kinetic energy of the ball at any point
   The kinetic energies of the ball at any point are equal because the velocities are the same.

34. Can the kinetic energy of a baseball ever be negative? (11.1)
   The kinetic energy of a baseball can never be negative because the kinetic energy depends on the square of the velocity, which is always positive.

35. Can the gravitational potential energy of a baseball ever be negative? Explain without using a formula. (11.1)
   The gravitational potential energy of a baseball can be negative if the height of the ball is lower than the reference level.

36. If a sprinter's velocity increases to three times the original velocity, by what factor does the kinetic energy increase? (11.1)
   The sprinter's kinetic energy increases by a factor of 9, because the velocity is squared.

37. What energy transformations take place when an athlete is pole-vaulting? (11.2)
   The pole-vaulter runs (kinetic energy) and bends the pole, thereby adding elastic potential energy to the pole. As he/she lifts his/her body, that kinetic and elastic potential energy is transferred into kinetic and gravitational potential energy. When he/she releases the pole, all of his/her energy is kinetic and gravitational potential energy.

38. The sport of pole-vaulting was drastically changed when the stiff, wooden poles were replaced by flexible, fiberglass poles. Explain why. (11.2)
   A flexible, fiberglass pole can store elastic potential energy because it can be bent easily. This energy can be released to push the pole-vaulter higher vertically. By contrast, the wooden pole does not store elastic potential energy, and the pole-vaulter's maximum height is limited by the direct conversion of kinetic energy to gravitational potential energy.

39. You throw a clay ball at a hockey puck on ice. The smashed clay ball and the hockey puck stick together and move slowly. (11.2)
   a. Is momentum conserved in the collision? Explain.
      The total momentum of the ball and the puck together is conserved in the collision because there are no unbalanced forces on this system.
      The total kinetic energy is not conserved. Part of it is lost in the smashing of the clay ball and the adhesion of the ball to the puck.

40. Draw energy bar graphs for the following processes. (11.2)
   a. An ice cube, initially at rest, slides down a frictionless slope.

   ![Energy Bar Graph](image)

   b. An ice cube, initially moving, slides up a frictionless slope and instantaneously comes to rest.

   ![Energy Bar Graph](image)
41. Describe the transformations from kinetic energy to potential energy and vice versa for a roller-coaster ride. (11.2)

On a roller-coaster ride, the car has mostly potential energy at the tops of the hills and mostly kinetic energy at the bottoms of the hills.

42. Describe how the kinetic energy and elastic potential energy are lost in a bouncing rubber ball. Describe what happens to the motion of the ball. (11.2)

On each bounce, some, but not all, of the ball’s kinetic energy is stored as elastic potential energy; the ball’s deformation dissipates the rest of the energy as thermal energy and sound. After the bounce, the stored elastic potential energy is released as kinetic energy. Due to the energy losses in the deformation, each subsequent bounce begins with a smaller amount of kinetic energy, and results in the ball reaching a lower height. Eventually, all of the ball’s energy is dissipated, and the ball comes to rest.

Applying Concepts
pages 306–307

43. The driver of a speeding car applies the brakes and the car comes to a stop. The system includes the car but not the road. Apply the work-energy theorem to the following situations.

a. The car’s wheels do not skid.
   If the car wheels do not skid, the brake surfaces rub against each other and do work that stops the car. The work that the brakes do is equal to the change in kinetic energy of the car. The brake surfaces heat up because the kinetic energy is transformed to thermal energy.

b. The brakes lock and the car’s wheels skid.
   If the brakes lock and the car wheels skid, the wheels rubbing on the road are doing the work that stops the car. The tire surfaces heat up, not the brakes. This is not an efficient way to stop a car, and it ruins the tires.

44. A compact car and a trailer truck are both traveling at the same velocity. Did the car engine or the truck engine do more work in accelerating its vehicle?

The trailer truck has more kinetic energy, \( KE = \frac{1}{2} mv^2 \), because it has greater mass than the compact car. Thus, according to the work-energy theorem, the truck’s engine must have done more work.

45. Catapults Medieval warriors used catapults to assault castles. Some catapults worked by using a tightly wound rope to turn the catapult arm. What forms of energy are involved in catapulting a rock to the castle wall?

Elastic potential energy is stored in the wound rope, which does work on the rock. The rock has kinetic and potential energy as it flies through the air. When it hits the wall, the inelastic collision causes most of the mechanical energy to be converted to thermal and sound energy and to do work breaking apart the wall structure. Some of the mechanical energy appears in the fragments thrown from the collision.

46. Two cars collide and come to a complete stop. Where did all of their energy go?

The energy went into bending sheet metal on the cars. Energy also was lost due to frictional forces between the cars and the tires, and in the form of thermal energy and sound.

47. During a process, positive work is done on a system, and the potential energy decreases. Can you determine anything about the change in kinetic energy of the system? Explain.

The work equals the change in the total mechanical energy, \( W = \Delta (KE + PE) \). If \( W \) is positive and \( \Delta PE \) is negative, then \( \Delta KE \) must be positive and greater than \( W \).

48. During a process, positive work is done on a system, and the potential energy increases. Can you tell whether the kinetic energy increased, decreased, or remained the same? Explain.
Chapter 11 continued

The work equals the change in the total mechanical energy, $W = \Delta(KE + PE)$. If $W$ is positive and $\Delta PE$ is positive, then you cannot say anything conclusive about $\Delta KE$.

49. Skating Two skaters of unequal mass have the same speed and are moving in the same direction. If the ice exerts the same frictional force on each skater, how will the stopping distances of their bodies compare?

The larger skater will have more kinetic energy. The kinetic energy of each skater will be dissipated by the negative work, $W = Fd$, done by the friction of the ice. Since the frictional forces are equal, the larger skater will go farther before stopping.

50. You swing a 55-g mass on the end of a 0.75-m string around your head in a nearly horizontal circle at constant speed, as shown in Figure 11-16.

![Figure 11-16](image)

a. How much work is done on the mass by the tension of the string in one revolution?

No work is done by the tension force on the mass because the tension is pulling perpendicular to the motion of the mass.

b. Is your answer to part a in agreement with the work-energy theorem? Explain.

This does not violate the work-energy theorem because the kinetic energy of the mass is constant; it is moving at a constant speed.

51. Give specific examples that illustrate the following processes.

a. Work is done on a system, thereby increasing kinetic energy with no change in potential energy.

pushing a hockey puck horizontally across ice; system consists of hockey puck only

b. Potential energy is changed to kinetic energy with no work done on the system.

dropping a ball; system consists of ball and Earth

c. Work is done on a system, increasing potential energy with no change in kinetic energy.

compressing the spring in a toy pistol; system consists of spring only

d. Kinetic energy is reduced, but potential energy is unchanged. Work is done by the system.

A car, speeding on a level track, brakes and reduces its speed.

52. Roller Coaster You have been hired to make a roller coaster more exciting. The owners want the speed at the bottom of the first hill doubled. How much higher must the first hill be built?

The hill must be made higher by a factor of 4.

53. Two identical balls are thrown from the top of a cliff, each with the same speed. One is thrown straight up, the other straight down. How do the kinetic energies and speeds of the balls compare as they strike the ground?

Even though the balls are moving in opposite directions, they have the same kinetic energy and potential energy when they are thrown. Therefore, they will have the same mechanical energy and speed when they hit the ground.
Mastering Problems

Unless otherwise directed, assume that air resistance is negligible.

11.1 The Many Forms of Energy

pages 307–308

Level 1

54. A 1600-kg car travels at a speed of 12.5 m/s. What is its kinetic energy?

\[
KE = \frac{1}{2}mv^2 = \frac{1}{2}(1600 \text{ kg})(12.5 \text{ m/s})^2
\]

\[
= 1.3 \times 10^5 \text{ J}
\]

55. A racing car has a mass of 1525 kg. What is its kinetic energy if it has a speed of 108 km/h?

\[
KE = \frac{1}{2}mv^2
\]

\[
= \frac{1}{2}(1525 \text{ kg})\left(\frac{108 \text{ km/h}}{3600 \text{ s/h}}\right)^2
\]

\[
= 6.86 \times 10^5 \text{ J}
\]

56. Shawn and his bike have a combined mass of 45.0 kg. Shawn rides his bike 1.80 km in 10.0 min at a constant velocity. What is Shawn’s kinetic energy?

\[
KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{d}{t}\right)^2
\]

\[
= \frac{1}{2}(45 \text{ kg})\left(\frac{1.80 \text{ km}}{10.0 \text{ min}}\right)^2
\]

\[
= 203 \text{ J}
\]

57. Tony has a mass of 45 kg and is moving with a speed of 10.0 m/s.

a. Find Tony’s kinetic energy.

\[
KE = \frac{1}{2}mv^2 = \frac{1}{2}(45 \text{ kg})(10.0 \text{ m/s})^2
\]

\[
= 2.3 \times 10^3 \text{ J}
\]

b. Tony’s speed changes to 5.0 m/s. Now what is his kinetic energy?

\[
KE = \frac{1}{2}mv^2 = \frac{1}{2}(45 \text{ kg})(5.0 \text{ m/s})^2
\]

\[
= 5.6 \times 10^2 \text{ J}
\]

c. What is the ratio of the kinetic energies in parts a and b? Explain.

\[
\frac{\frac{1}{2}(mv_1^2)}{\frac{1}{2}(mv_2^2)} = \frac{v_1^2}{v_2^2} = \frac{(10.0)^2}{(5.0)^2} = \frac{4}{1}
\]

Twice the velocity gives four times the kinetic energy. The kinetic energy is proportional to the square of the velocity.

58. Katia and Angela each have a mass of 45 kg, and they are moving together with a speed of 10.0 m/s.

a. What is their combined kinetic energy?

\[
KE_c = \frac{1}{2}mv^2 = \frac{1}{2}(m_K + m_A)v^2
\]

\[
= \frac{1}{2}(45 \text{ kg} + 45 \text{ kg})(10.0 \text{ m/s})^2
\]

\[
= 4.5 \times 10^3 \text{ J}
\]

b. What is the ratio of their combined mass to Katia’s mass?

\[
\frac{m_K + m_A}{m_K} = \frac{45 \text{ kg} + 45 \text{ kg}}{45 \text{ kg}}
\]

\[
= \frac{2}{1}
\]

c. What is the ratio of their combined kinetic energy to Katia’s kinetic energy? Explain.

\[
KE_K = \frac{1}{2}m_Kv^2 = \frac{1}{2}(45 \text{ kg})(10.0 \text{ m/s})^2
\]

\[
= 2.3 \times 10^3 \text{ J}
\]

\[
\frac{KE_c}{KE_K} = \frac{\frac{1}{2}(m_K + m_A)v^2}{\frac{1}{2}m_Kv^2} = \frac{m_K + m_A}{m_K}
\]

\[
= \frac{2}{1}
\]

The ratio of their combined kinetic energy to Katia’s kinetic energy is the same as the ratio of their combined mass to Katia’s mass. Kinetic energy is proportional to mass.
59. **Train** In the 1950s, an experimental train, which had a mass of \(2.50 \times 10^4\) kg, was powered across a level track by a jet engine that produced a thrust of \(5.00 \times 10^5\) N for a distance of 509 m.

a. Find the work done on the train.

\[
W = Fd = (5.00 \times 10^5\ \text{N})(509\ \text{m}) = 2.55 \times 10^8\ \text{J}
\]

b. Find the change in kinetic energy.

\[
\Delta KE = W = 2.55 \times 10^8\ \text{J}
\]

c. Find the final kinetic energy of the train if it started from rest.

\[
\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + W
\]

\[
= \frac{1}{2}(2.50 \times 10^4\ \text{kg}) \times 2.55 \times 10^8\ \text{J}
\]

\[
= 2.04 \times 10^9\ \text{m}^2/\text{s}^2 = 143\ \text{m/s}
\]

d. Find the final speed of the train if there had been no friction.

\[
\Delta KE = KE_f - KE_i
\]

\[
= \frac{1}{2}m(v_f^2 - v_i^2)
\]

\[
= \frac{1}{2}(14.700\ \text{N})(25.0\ \text{m/s})^2 - (7.50\ \text{m/s})^2
\]

\[
= -345\ \text{J}
\]

60. **Car Brakes** A 14,700-N car is traveling at 25 m/s. The brakes are applied suddenly, and the car slides to a stop, as shown in **Figure 11-17**. The average braking force between the tires and the road is 7100 N. How far will the car slide once the brakes are applied?

\[
W = Fd = \frac{1}{2}mv^2
\]

\[
W = Fd = \frac{1}{2} \times 14,700\ \text{N} \times (25\ \text{m/s})^2
\]

\[
= 10.0\ \text{N}\ \frac{1}{2} \times 7100\ \text{N} = 34.5\ \text{m}
\]

61. A 15.0-kg cart is moving with a velocity of 7.50 m/s down a level hallway. A constant force of 10.0 N acts on the cart, and its velocity becomes 3.20 m/s.

a. What is the change in kinetic energy of the cart?

\[
\Delta KE = KE_f - KE_i = \frac{1}{2}m(v_f^2 - v_i^2)
\]

\[
= \frac{1}{2}(15.0\ \text{kg})(3.20\ \text{m/s})^2 - (7.50\ \text{m/s})^2
\]

\[
= -345\ \text{J}
\]

b. How much work was done on the cart?

\[
W = \Delta KE = -345\ \text{J}
\]

c. How far did the cart move while the force acted?

\[
W = Fd
\]

\[
so\ d = \frac{W}{F} = \frac{-345\ \text{J}}{-10.0\ \text{N}} = 34.5\ \text{m}
\]

62. How much potential energy does DeAnna with a mass of 60.0 kg, gain when she climbs a gymnasium rope a distance of 3.5 m?

\[
PE = mgh
\]

\[
= (60.0\ \text{kg})(9.80\ \text{m/s}^2)(3.5\ \text{m})
\]

\[
= 2.1 \times 10^3\ \text{J}
\]

63. **Bowling** A 6.4-kg bowling ball is lifted 2.1 m into a storage rack. Calculate the increase in the ball’s potential energy.

\[
PE = mgh
\]

\[
= (6.4\ \text{kg})(9.80\ \text{m/s}^2)(2.1\ \text{m})
\]

\[
= 1.3 \times 10^2\ \text{J}
\]
64. Mary weighs 505 N. She walks down a flight of stairs to a level 5.50 m below her starting point. What is the change in Mary’s potential energy?

\[ PE = mg\Delta h = F_g \Delta h = (505 \text{ N})(-5.50 \text{ m}) = -2.78 \times 10^3 \text{ J} \]

65. Weightlifting A weightlifter raises a 180-kg barbell to a height of 1.95 m. What is the increase in the potential energy of the barbell?

\[ PE = mgh = (180 \text{ kg})(9.80 \text{ m/s}^2)(1.95 \text{ m}) = 3.4 \times 10^3 \text{ J} \]

66. A 10.0-kg test rocket is fired vertically from Cape Canaveral. Its fuel gives it a kinetic energy of 1960 J by the time the rocket engine burns all of the fuel. What additional height will the rocket rise?

\[ PE = mgh = KE \Rightarrow h = \frac{KE}{mg} = \frac{1960}{(10.0 \text{ kg})(9.80 \text{ m/s}^2)} = 20.0 \text{ m} \]

67. Antwan raised a 12.0-N physics book from a table 75 cm above the floor to a shelf 2.15 m above the floor. What was the change in the potential energy of the system?

\[ PE = mg\Delta h = F_g \Delta h = F_g(h_f - h_i) = (12.0 \text{ N})(2.15 \text{ m} - 0.75 \text{ m}) = 17 \text{ J} \]

68. A hallway display of energy is constructed in which several people pull on a rope that lifts a block 1.00 m. The display indicates that 1.00 J of work is done. What is the mass of the block?

\[ W = PE = mgh \Rightarrow m = \frac{W}{gh} = \frac{1.00 \text{ J}}{(9.80 \text{ m/s}^2)(1.00 \text{ m})} = 0.102 \text{ kg} \]

69. Tennis It is not uncommon during the serve of a professional tennis player for the racket to exert an average force of 150.0 N on the ball. If the ball has a mass of 0.060 kg and is in contact with the strings of the racket, as shown in Figure 11-18, for 0.030 s, what is the kinetic energy of the ball as it leaves the racket? Assume that the ball starts from rest.

\[ Ft = m\Delta v = mv_f - mv_i \Rightarrow v_i = 0 \]

so \[ v_f = \frac{Ft}{m} = \frac{(150.0 \text{ N})(3.0 \times 10^{-2} \text{ s})}{6.0 \times 10^{-2} \text{ kg}} = 75 \text{ m/s} \]

\[ KE_f = \frac{1}{2}mv^2 = \frac{1}{2}(6.0 \times 10^{-2} \text{ kg})(75 \text{ m/s})^2 = 1.7 \times 10^2 \text{ J} \]

70. Pam, wearing a rocket pack, stands on frictionless ice. She has a mass of 45 kg. The rocket supplies a constant force for 22.0 m, and Pam acquires a speed of 62.0 m/s.

a. What is the magnitude of the force?

\[ \Delta KE_i = \frac{1}{2}mv^2 = \frac{1}{2}(45 \text{ kg})(62.0 \text{ m/s})^2 = 8.6 \times 10^4 \text{ J} \]

b. What is Pam’s final kinetic energy?

Work done on Pam equals her change in kinetic energy.

\[ W = Fd = \Delta KE = KE_f - KE_i \Rightarrow KE_i = 0 \text{ J} \]

So, \[ F = \frac{KE_f}{d} = \frac{8.6 \times 10^4 \text{ J}}{22.0 \text{ m}} = 3.9 \times 10^3 \text{ N} \]
71. **Collision** A $2.00 \times 10^3$-kg car has a speed of 12.0 m/s. The car then hits a tree. The tree doesn’t move, and the car comes to rest, as shown in Figure 11-19.

- **Figure 11-19**
  - a. Find the change in kinetic energy of the car.
    \[
    \Delta KE = KE_f - KE_i = \frac{1}{2} m(v_f^2 - v_i^2) = \frac{1}{2} (2.00 \times 10^3 \text{ kg})((0.0 \text{ m/s})^2 - (12.0 \text{ m/s})^2) = -1.44 \times 10^5 \text{ J}
    \]
  - b. Find the amount of work done as the front of the car crashes into the tree.
    \[
    W = \Delta KE = -1.44 \times 10^5 \text{ J}
    \]
  - c. Find the size of the force that pushed in the front of the car by 50.0 cm.
    \[
    W = Fd \\
    \text{so } F = \frac{W}{d} = \frac{-1.44 \times 10^5 \text{ J}}{0.500 \text{ m}} = -2.88 \times 10^5 \text{ N}
    \]

72. A constant net force of 410 N is applied upward to a stone that weighs 32 N. The upward force is applied through a distance of 2.0 m, and the stone is then released. To what height, from the point of release, will the stone rise?

- Work done:
  \[
  W = Fd = (410 \text{ N})(2.0 \text{ m}) = 8.2 \times 10^2 \text{ J}
  \]
- Use work and potential energy formula:
  \[
  \Delta h = \frac{W}{mg} = \frac{8.2 \times 10^2 \text{ J}}{32 \text{ N}} = 26 \text{ m}
  \]

73. A 98.0-N sack of grain is hoisted to a storage room 50.0 m above the ground floor of a grain elevator.

- a. How much work was done?
  \[
  W = \Delta PE = mg\Delta h = F_\text{g}\Delta h = (98.0 \text{ N})(50.0 \text{ m}) = 4.90 \times 10^3 \text{ J}
  \]
- b. What is the increase in potential energy of the sack of grain at this height?
  \[
  \Delta PE = W = 4.90 \times 10^3 \text{ J}
  \]
- c. The rope being used to lift the sack of grain breaks just as the sack reaches the storage room. What kinetic energy does the sack have just before it strikes the ground floor?
  \[
  KE = \Delta PE = 4.90 \times 10^3 \text{ J}
  \]

74. A 20-kg rock is on the edge of a 100-m cliff, as shown in Figure 11-20.

- a. What potential energy does the rock possess relative to the base of the cliff?
  \[
  PE = mgh = (20 \text{ kg})(9.80 \text{ m/s}^2)(100 \text{ m}) = 2 \times 10^4 \text{ J}
  \]
- b. The rock falls from the cliff. What is its kinetic energy just before it strikes the ground?
  \[
  KE = \Delta PE = 2 \times 10^4 \text{ J}
  \]
- c. What speed does the rock have as it strikes the ground?
  \[
  KE = \frac{1}{2} mv^2 \\
  v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{(2)(2 \times 10^4 \text{ J})}{20 \text{ kg}}} = 40 \text{ m/s}
  \]
75. **Archery** An archer puts a 0.30-kg arrow to the bowstring. An average force of 201 N is exerted to draw the string back 1.3 m.

   a. Assuming that all the energy goes into the arrow, with what speed does the arrow leave the bow?

   Work done on the string increases the string’s elastic potential energy.

   \[ W = \Delta PE = Fd \]

   All of the stored potential energy is transformed to the arrow’s kinetic energy.

   \[ KE = \frac{1}{2} mv^2 = \Delta PE = Fd \]

   \[ v^2 = \frac{2Fd}{m} \]

   \[ v = \sqrt{\frac{2Fd}{m}} = \sqrt{\frac{(2)(201 N)(1.3 m)}{(0.30 \text{ kg})(9.80 \text{ m/s}^2)}} \]

   \[ = 42 \text{ m/s} \]

   b. If the arrow is shot straight up, how high does it rise?

   The change in the arrow’s potential energy equals the work done to pull the string.

   \[ \Delta PE = mg\Delta h = Fd \]

   \[ \Delta h = \frac{Fd}{mg} = \frac{(201 N)(1.3 m)}{(0.30 \text{ kg})(9.80 \text{ m/s}^2)} \]

   \[ = 89 \text{ m} \]

76. A 2.0-kg rock that is initially at rest loses 407 J of potential energy while falling to the ground. Calculate the kinetic energy that the rock gains while falling. What is the rock’s speed just before it strikes the ground?

   \[ PE_i + KE_i = PE_f + KE_f \]

   \[ KE_i = 0 \]

   So,

   \[ KE_f = PE_f - PE_i = 407 \text{ J} \]

   \[ KE_f = \frac{1}{2} mv_f^2 \]

   \[ v_f^2 = \frac{2KE_f}{m} \]

   \[ v_f = \sqrt{\frac{2KE_f}{m}} = \sqrt{\frac{(2)(407 \text{ J})}{(2.0 \text{ kg})}} \]

   \[ = 2.0 \times 10^1 \text{ m/s} \]

77. A physics book of unknown mass is dropped 4.50 m. What speed does the book have just before it hits the ground?

   \[ KE = PE \]

   \[ \frac{1}{2} mv^2 = mgh \]

   The mass of the book divides out, so

   \[ \frac{1}{2} v^2 = gh \]

   \[ v = \sqrt{2gh} = \sqrt{2(9.80 \text{ m/s}^2)(4.50 \text{ m})} \]

   \[ = 9.39 \text{ m/s} \]

78. **Railroad Car** A railroad car with a mass of \(5.0 \times 10^3 \text{ kg}\) collides with a stationary railroad car of equal mass. After the collision, the two cars lock together and move off at 4.0 m/s, as shown in **Figure 11-21**.

   \[ m = 5.0 \times 10^5 \text{ kg} \]

   \[ v = 4.0 \text{ m/s} \]

   a. Before the collision, the first railroad car was moving at 8.0 m/s. What was its momentum?

   \[ mv = (5.0 \times 10^5 \text{ kg})(8.0 \text{ m/s}) \]

   \[ = 4.0 \times 10^6 \text{ kg-m/s} \]

   b. What was the total momentum of the two cars after the collision?

   Because momentum is conserved, it must be \(4.0 \times 10^6 \text{ kg-m/s}\)

   c. What were the kinetic energies of the two cars before and after the collision?

   Before the collision:

   \[ KE_i = \frac{1}{2} mv_i^2 \]

   \[ = \frac{1}{2} (5.0 \times 10^5 \text{ kg})(8.0 \text{ m/s})^2 \]

   \[ = 1.6 \times 10^7 \text{ J} \]

   After the collision:

   \[ KE_f = \frac{1}{2} mv^2 \]
Chapter 11 continued

\[ KE = \frac{1}{2} mv^2 = \frac{1}{2} \left( \frac{(27.8 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} \right) = 39.4 \text{ m} \]

Level 2

79. From what height would a compact car have to be dropped to have the same kinetic energy that it has when being driven at \(1.00 \times 10^2 \text{ km/h} \)?

\[ v = \sqrt{\frac{2gh}{m}} = \sqrt{\frac{(27.8 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)}} = 39.4 \text{ m} \]

80. Kelli weighs 420 N, and she is sitting on a playground swing that hangs 0.40 m above the ground. Her mom pulls the swing back and releases it when the seat is 1.00 m above the ground.

a. How fast is Kelli moving when the swing passes through its lowest position?

\[ \Delta KE = \Delta m \Delta v = m \Delta (v_f - v_i) \]

\[ \Delta m = \frac{1}{2} m(v_f^2 - v_i^2) = \frac{1}{2} mv_f^2 \]

By conservation of mechanical energy:

\[ \Delta KE + \Delta PE = 0 \]

\[ mg(h_f - h_i) + \frac{1}{2} mv_f^2 = 0 \]

\[ v_i = \sqrt{\frac{2g(h_i - h_f)}{m}} = \sqrt{\frac{(2)(9.80 \text{ m/s}^2)(1.00 \text{ m} - 0.40 \text{ m})}{28 \text{ kg}}} = 3.4 \text{ m/s} \]

b. If Kelli moves through the lowest point at 2.0 m/s, how much work was done on the swing by friction?

The work done by friction equals the change in mechanical energy.

\[ W = \Delta PE - \Delta KE \]

\[ = mg(h_f - h_i) + \frac{1}{2} mv_f^2 \]

\[ = (420 \text{ N})(0.40 \text{ m} - 1.00 \text{ m}) + \frac{1}{2} (420 \text{ N})(2.0 \text{ m/s})^2 \]

\[ = -1.7 \times 10^2 \text{ J} \]

81. Hakeem throws a 10.0-g ball straight down from a height of 2.0 m. The ball strikes the floor at a speed of 7.5 m/s. What was the initial speed of the ball?

\[ KE_i = KE_f + PE_i \]

\[ \frac{1}{2} mv_i^2 = \frac{1}{2} mv_f^2 + mgh \]

the mass of the ball divides out, so

\[ v_i^2 = v_f^2 - 2gh \]

\[ v_i = \sqrt{v_f^2 - 2gh} = \sqrt{(7.5 \text{ m/s})^2 - (2)(9.80 \text{ m/s}^2)(2.0 \text{ m})} = 4.1 \text{ m/s} \]

82. Slide Lorena’s mass is 28 kg. She climbs the 4.8-m ladder of a slide and reaches a velocity of 3.2 m/s at the bottom of the slide. How much work was done by friction on Lorena?

The work done by friction on Lorena equals the change in her mechanical energy.

\[ W = \Delta PE + \Delta KE \]

\[ = mg(h_f - h_i) + \frac{1}{2} m(v_f^2 - v_i^2) \]

\[ = (28 \text{ kg})(9.80 \text{ m/s}^2)(0.0 \text{ m} - 4.8 \text{ m}) + \frac{1}{2} (28 \text{ kg})((3.2 \text{ m/s})^2 - (0.0 \text{ m/s})^2) \]

\[ = -1.2 \times 10^3 \text{ J} \]

83. A person weighing 635 N climbs up a ladder to a height of 5.0 m. Use the person and Earth as the system.
a. Draw energy bar graphs of the system before the person starts to climb the ladder and after the person stops at the top. Has the mechanical energy changed? If so, by how much?

Yes. The mechanical energy has changed, increase in potential energy of \( (635 \text{ N})(5.0 \text{ m}) = 3200 \text{ J} \).

b. Where did this energy come from?

from the internal energy of the person

Mixed Review

pages 309–310

Level 1

84. Suppose a chimpanzee swings through the jungle on vines. If it swings from a tree on a 13-m-long vine that starts at an angle of 45°, what is the chimp’s velocity when it reaches the ground?

The chimpanzee's initial height is

\[ h = (13 \text{ m})(1 - \cos 45°) = 3.8 \text{ m} \]

Conservation of mechanical energy:

\[ \Delta PE + \Delta KE = 0 \]

\[ mg(h_i - h_i) + \frac{1}{2} m(v_i^2 - v_f^2) = 0 \]

\[-mgh_i + \frac{1}{2} mv_i^2 = 0 \]

\[ v_i = \sqrt{2gh_i} = \sqrt{2(9.80 \text{ m/s}^2)(3.8 \text{ m})} \]

\[ = 8.6 \text{ m/s} \]

85. An 0.80-kg cart rolls down a frictionless hill of height 0.32 m. At the bottom of the hill, the cart rolls on a flat surface, which exerts a frictional force of 2.0 N on the cart. How far does the cart roll on the flat surface before it comes to a stop?

\[ E = mgh = W = Fd \]

\[ d = \frac{mgh}{F} = \frac{(0.80 \text{ kg})(9.80 \text{ m/s}^2)(0.32 \text{ m})}{2.0 \text{ N}} \]

\[ = 1.3 \text{ m} \]

86. **High Jump** The world record for the men’s high jump is about 2.45 m. To reach that height, what is the minimum amount of work that a 73.0-kg jumper must exert in pushing off the ground?

\[ W = \Delta E = mgh \]

\[ = (73.0 \text{ kg})(9.80 \text{ m/s}^2)(2.45 \text{ m}) \]

\[ = 1.75 \text{ kJ} \]

87. A stuntwoman finds that she can safely break her fall from a one-story building by landing in a box filled to a 1-m depth with foam peanuts. In her next movie, the script calls for her to jump from a five-story building. How deep a box of foam peanuts should she prepare?

Assume that the foam peanuts exert a constant force to slow him down, \( W = Fd = E = mgh \). If the height is increased five times, then the depth of the foam peanuts also should be increased five times to 5 m.

Level 2

88. **Football** A 110-kg football linebacker has a head-on collision with a 150-kg defensive end. After they collide, they come to a complete stop. Before the collision, which player had the greater momentum and which player had the greater kinetic energy?

The momentum after the collision is zero; therefore, the two players had equal and opposite momenta before the collision. That is,

\[ p_{\text{linebacker}} = m_{\text{linebacker}} v_{\text{linebacker}} \]

\[ p_{\text{end}} = m_{\text{end}} v_{\text{end}} \]

After the collision, each had zero energy. The energy loss for each player was

\[ \frac{1}{2} mv^2 = \frac{1}{2} \left( \frac{m^2 v^2}{m} \right) = \frac{p^2}{2m} \]

Because the momenta were equal but

\[ m_{\text{linebacker}} < m_{\text{end}} \]

the linebacker lost more energy.

89. A 2.0-kg lab cart and a 1.0-kg lab cart are held together by a compressed spring. The lab carts move at 2.1 m/s in one direction. The spring suddenly becomes uncompressed and pushes the two lab carts apart. The 2-kg
Chapter 11 continued

lab cart comes to a stop, and the 1.0-kg lab cart moves ahead. How much energy did the spring add to the lab carts?

\[ E_i = \frac{1}{2} m v^2 = \frac{1}{2} (2.0 \text{ kg} + 1.0 \text{ kg})(2.1 \text{ m/s})^2 \]

\[ = 6.6 \text{ J} \]

\[ p_i = m v = (2.0 \text{ kg} + 1.0 \text{ kg})(2.1 \text{ m/s}) \]

\[ = 6.3 \text{ kg-m/s} = p_i = (1.0 \text{ kg})v_i \]

so, \( v_i = 6.3 \text{ m/s} \)

\[ E_i = \frac{1}{2} m v_i^2 = \frac{1}{2} (1.0 \text{ kg})(6.3 \text{ m/s})^2 = 19.8 \text{ J} \]

\[ \Delta E = 19.8 \text{ J} - 6.6 \text{ J} = 13.2 \text{ J} \]

13.2 J was added by the spring.

90. A 55.0-kg scientist roping through the top of a tree in the jungle sees a lion about to attack a tiny antelope. She quickly swings down from her 12.0-m-high perch and grabs the antelope (21.0 kg) as she swings. They barely swing back up to a tree limb out of reach of the lion. How high is this tree limb?

\[ E_i = m_B g h \]

The velocity of the botanist when she reaches the ground is

\[ E_i = \frac{1}{2} m_B v^2 = m_B g h \]

\[ v = \sqrt{\frac{2E_i}{m_B}} = \sqrt{\frac{2m_B g h}{m_B}} = \sqrt{2g h} \]

Momentum is conserved when the botanist grabs the antelope.

\[ m_B v = (m_B + m_A) v_i \]

so, \( v_i = \frac{m_B v}{m_B + m_A} = \left( \frac{m_B}{m_B + m_A} \right) \sqrt{2g h} \)

The final energy of the two is

\[ E_f = \frac{1}{2} (m_B + m_A) v_i^2 \]

\[ = \frac{1}{2} (m_B + m_A) \left( \frac{m_B}{m_B + m_A} \right)^2 (2g h) \]

\[ = (m_B + m_A) g h_i \]

So, \( h_i = \left( \frac{m_B}{m_B + m_A} \right)^2 h \)

\[ = \left( \frac{55.0 \text{ kg}}{55.0 \text{ kg} + 21.0 \text{ kg}} \right)^2 (12.0 \text{ m}) \]

\[ = 6.28 \text{ m} \]

91. An 0.80-kg cart rolls down a 30.0° hill from a vertical height of 0.50 m as shown in Figure 11-22. The distance that the cart must roll to the bottom of the hill is 0.50 m/sin 30.0° = 1.0 m. The surface of the hill exerts a frictional force of 5.0 N on the cart. Does the cart roll to the bottom of the hill?

\[ m = 0.80 \text{ kg} \]

\[ F = 5.0 \text{ N} \]

\[ E_i = m g h = (0.80 \text{ kg})(9.80 \text{ m/s}^2)(0.50 \text{ m}) \]

\[ = 3.9 \text{ J} \]

The work done by friction over 1.0 m would be

\[ W = F d = (5.0 \text{ N})(1.0 \text{ m}) = 5.0 \text{ J.} \]

The work done by friction is greater than the energy of the cart. The cart would not reach the bottom of the hill.

Level 3

92. Object A, sliding on a frictionless surface at 3.2 m/s, hits a 2.0-kg object, B, which is motionless. The collision of A and B is completely elastic. After the collision, A and B move away from each other at equal and opposite speeds. What is the mass of object A?

\[ p_i = m_A v_i + 0 \]

\[ p_i = m_A (-v_2) + m_B v_2 \]

\[ p_i = p_i \text{ (conservation of momentum)} \]

therefore, \( m_A v_1 = m_A (-v_2) + m_B v_2 \)

\[ (m_B - m_A) v_2 = m_A v_1 \]

\[ v_2 = \frac{(m_A v_1)}{(m_B - m_A)} \]

\[ E_i = \frac{1}{2} m_A v_1^2 \]

\[ E_f = \frac{1}{2} m_A v_2^2 + \frac{1}{2} m_B v_2^2 \]
Chapter 11 continued

\[ E_f = \frac{1}{2} (m_A + m_B)v_f^2 \]
\[ = \frac{1}{2} (m_A + m_B)\left(\frac{(m_A v_f)}{(m_B - m_A)}\right)^2 \]

\[ E_i = E_i \text{ (conservation of energy in elastic collision)} \]

\[ \frac{1}{2} m_A v_A^2 = \frac{1}{2} (m_A + m_B) \]
\[ \left(\frac{(m_A v_f)}{(m_B - m_A)}\right)^2 \]

After cancelling out common factors,

\[ (m_A + m_B)m_A = (m_B - m_A)^2 = \]
\[ m_B^2 - 2m_A m_B + m_A^2 \]
\[ m_A = \frac{m_B}{3} = \frac{2.00 \text{ kg}}{3} = 0.67 \text{ kg} \]

93. **Hockey** A 90.0-kg hockey player moving at 5.0 m/s collides head-on with a 110-kg hockey player moving at 3.0 m/s in the opposite direction. After the collision, they move off together at 1.0 m/s. How much energy was lost in the collision?

**Before:**

\[ E = \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2 \]
\[ = \frac{1}{2} (90.0 \text{ kg})(5.0 \text{ m/s})^2 + \]
\[ \frac{1}{2} (110 \text{ kg})(3.0 \text{ m/s})^2 \]
\[ = 1.6 \times 10^3 \text{ J} \]

**After:**

\[ E = \frac{1}{2} (m + m)v_f^2 \]
\[ = \frac{1}{2} (200.0 \text{ kg})(1.0 \text{ m/s})^2 \]
\[ = 1.0 \times 10^2 \text{ J} \]

**Energy loss**

\[ = 1.6 \times 10^3 \text{ J} - 1.0 \times 10^2 \text{ J} \]
\[ = 1.5 \times 10^3 \text{ J} \]

**Thinking Critically**

94. **Apply Concepts** A golf ball with a mass of 0.046 kg rests on a tee. It is struck by a golf club with an effective mass of 0.220 kg and a speed of 44 m/s. Assuming that the collision is elastic, find the speed of the ball when it leaves the tee.

From the conservation of momentum,

\[ m_c v_{c1} = m_c v_{c2} + m_b v_{b2} \]

Solve for \( v_{c2} \):

\[ v_{c2} = v_{c1} - \frac{m_b v_{b2}}{m_c} \]

From conservation of energy,

\[ \frac{1}{2} m_c v_{c1}^2 = \frac{1}{2} m_c v_{c2}^2 + \frac{1}{2} m_b v_{b2}^2 \]

Multiply by two and substitute to get:

\[ m_c v_{c1}^2 = m_c (v_{c1} - \frac{m_b v_{b2}}{m_c})^2 + m_b v_{b2}^2 \]

or \( m_c v_{c1}^2 = m_c v_{c1}^2 - 2m_b v_{c1} v_{b2} + m_b v_{b2}^2 \)

\[ \frac{m_b^2 v_{b2}^2}{m_c} + m_b v_{b2}^2 \]

Simplify and factor:

\[ 0 = (m_b v_{b2}) \left(-2v_{c1} + \frac{m_b^2 v_{b2}}{m_c} + v_{b2}\right) \]

\( m_b v_{b2} = 0 \) or

\[ -2v_{c1} + (\frac{m_b}{m_c} + 1)v_{b2} = 0 \]

Ignoring the solution \( v_{b2} = 0 \), then

\[ v_{b2} = \frac{2v_{c1}}{\left(\frac{m_b}{m_c} + 1\right)} \]

\[ = \frac{2(44 \text{ m/s})}{\left(\frac{0.046 \text{ kg}}{0.220 \text{ kg}} + 1\right)} = 73 \text{ m/s} \]

95. **Apply Concepts** A fly hitting the windshield of a moving pickup truck is an example of a collision in which the mass of one of the objects is many times larger than the other. On the other hand, the collision of two billiard balls is one in which the masses of both objects are the same. How is energy transferred in these collisions? Consider an elastic collision in which billiard ball \( m_1 \) has velocity \( v_1 \) and ball \( m_2 \) is motionless.

**a.** If \( m_1 = m_2 \), what fraction of the initial energy is transferred to \( m_2 \)?

**If \( m_1 = m_2 \), we know that \( m_1 \) will be at rest after the collision and \( m_2 \) will move with velocity \( v_2 \). All of the energy will be transferred to \( m_2 \).**
Chapter 11 continued

b. If \( m_1 \gg m_2 \), what fraction of the initial energy is transferred to \( m_2 \)?

If \( m_1 \gg m_2 \), we know that the motion of \( m_1 \) will be unaffected by the collision and that the energy transfer to \( m_2 \) will be minimal.

c. In a nuclear reactor, neutrons must be slowed down by causing them to collide with atoms. (A neutron is about as massive as a proton.) Would hydrogen, carbon, or iron atoms be more desirable to use for this purpose?

The best way to stop a neutron is to have it hit a hydrogen atom, which has about the same mass as the neutron.

96. Analyze and Conclude In a perfectly elastic collision, both momentum and mechanical energy are conserved. Two balls, with masses \( m_A \) and \( m_B \), are moving toward each other with speeds \( v_A \) and \( v_B \), respectively. Solve the appropriate equations to find the speeds of the two balls after the collision.

conservation of momentum

(1) \( m_A v_A + m_B v_B = m_A v_{A2} + m_B v_{B2} \)

(2) \( m_A (v_A - v_{A2}) = -m_B (v_B - v_{B2}) \)

conservation of energy

(3) \( \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A v_{A2}^2 + \frac{1}{2} m_B v_{B2}^2 \)

Divide equation (3) by (2) to obtain

(4) \( v_A + v_{A2} = v_B + v_{B2} \)

Solve equation (1) for \( v_{A2} \) and \( v_{B2} \)

\( v_{A2} = v_A + \frac{m_B}{m_A} (v_B - v_{B2}) \)

\( v_{B2} = v_B + \frac{m_A}{m_B} (v_A - v_{A2}) \)

Substitute into (4) and solve for \( v_{B2} \) and \( v_{A2} \)

\( v_A + v_{A2} + \frac{m_B}{m_A} (v_B - v_{B2}) = v_B + v_{B2} \)

\( 2m_A v_A + m_B v_B - m_B v_{B2} = m_A v_{A2} + m_B v_{B2} \)

\( v_{B2} = \left( \frac{2m_A}{m_A + m_B} \right) v_A + \left( \frac{m_B - m_A}{m_A + m_B} \right) v_B \)

\( v_{A1} + v_{A2} = v_B + v_{B1} + \frac{m_A}{m_B} (v_A - v_{A2}) \)

\( m_B v_A + m_B v_{A2} = 2m_B v_{B1} + m_A v_{A1} - m_A v_{A2} \)

\( v_{A2} = \left( \frac{m_A - m_B}{m_A + m_B} \right) v_A + \left( \frac{2m_B}{m_A + m_B} \right) v_B \)
97. **Analyze and Conclude**  A 25-g ball is fired with an initial speed of $v_1$ toward a 125-g ball that is hanging motionless from a 1.25-m string. The balls have a perfectly elastic collision. As a result, the 125-g ball swings out until the string makes an angle of $37.0^\circ$ with the vertical. What is $v_1$?

**Object 1** is the incoming ball. **Object 2** is the one attached to the string. In the collision, momentum is conserved.

$$p_{1i} = p_{1f} + p_{2f} \text{ or } m_1v_{1i} = m_1v_{1f} + m_2v_{2f}$$

In the collision, kinetic energy is conserved.

$$\frac{1}{2}m_1v_{1i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

$$m_1v_{1i}^2 = m_1v_{1f}^2 + m_2v_{2f}^2$$

$$(m_1v_{1i})^2 = (m_1v_{1f})^2 + (m_2v_{2f})^2$$

$$\frac{m_1^2v_{1i}^2}{m_1} = m_1^2v_{1f}^2 + m_2^2v_{2f}^2$$

$$\frac{p_{1i}^2}{m_1} = p_{1f}^2 + p_{2f}^2$$

$$p_{1i}^2 = p_{1f}^2 + \left(\frac{m_1}{m_2}\right)p_{2f}^2$$

We don’t care about $v_{1f}$, so get rid of $p_{1i}$ using $p_{1i} = p_{1i} - p_{2f}$

$$p_{1i}^2 = (p_{1i} - p_{2f})^2 + \frac{m_1}{m_2}p_{2f}^2$$

$$p_{1i}^2 = p_{1i}^2 - 2p_{1i}p_{2f} + p_{2f}^2 + \frac{m_1}{m_2}p_{2f}^2$$

$$2p_{1i}p_{2f} = \left(1 + \frac{m_1}{m_2}\right)p_{2f}^2$$

$$p_{1i} = \left(\frac{1}{2}\right)\left(1 + \frac{m_1}{m_2}\right)p_{2f}$$

$$m_1v_{1i} = \left(\frac{1}{2}\right)(m_2 + m_1)v_{2f}$$

$$v_{1i} = \left(\frac{1}{2}\right)\left(\frac{m_2}{m_1} + 1\right)v_{2f}$$

Now consider the pendulum.

$$\frac{1}{2}m_2v_{2f}^2 = m_2gh$$

or $v_{2f} = \sqrt{2gh}$

where $h = L(1 - \cos \theta)$

Thus, $v_{2f} = \sqrt{2gL(1 - \cos \theta)}$

$$v_{2f} = \sqrt{(2)(9.80 \text{ m/s}^2)(1.25 \text{ m})(1 - \cos 37.0^\circ)}$$

$$= 2.22 \text{ m/s}$$
Writing in Physics

98. All energy comes from the Sun. In what forms has this solar energy come to us to allow us to live and to operate our society? Research the ways that the Sun’s energy is turned into a form that we can use. After we use the Sun’s energy, where does it go? Explain.

The Sun produces energy through nuclear fusion and releases that energy in the form of electromagnetic radiation, which is transferred through the vacuum of space to Earth. Earth absorbs that electromagnetic radiation in its atmosphere, land, and oceans in the form of thermal energy or heat. Part of the visible radiation also is converted by plants into chemical energy through photosynthesis. There are several other chemical reactions mediated by sunlight, such as ozone production. The energy then is transferred into various forms, some of which are the chemical processes that allow us to digest food and turn it into chemical energy to build tissues, to move, and to think. In the end, after we have used the energy, the remainder is dispersed as electromagnetic radiation back into the universe.

99. All forms of energy can be classified as either kinetic or potential energy. How would you describe nuclear, electric, chemical, biological, solar, and light energy, and why? For each of these types of energy, research what objects are moving and how energy is stored in those objects.

Potential energy is stored in the binding of the protons and neutrons in the nucleus. The energy is released when a heavy nucleus is broken into smaller pieces (fission) or when very small nuclei are combined to make bigger nuclei (fusion). In the same way, chemical potential energy is stored when atoms are combined to make molecules and released when the molecules are broken up or rearranged. Separation of electric charges produces electric potential energy, as in a battery. Electric potential energy is converted to kinetic energy in the motion of electric charges in an electric current when a conductive path, or circuit, is provided. Biological processes are all chemical, and thus, biological energy is just a form of chemical energy. Solar energy is fusion energy converted to electromagnetic radiation. (See the answer to the previous question.) Light is a wave form of electromagnetic energy whose frequency is in a range detectible by the human eye.

Cumulative Review

100. A satellite is placed in a circular orbit with a radius of $1.0 \times 10^7$ m and a period of $9.9 \times 10^3$ s. Calculate the mass of Earth.

**Hint:** Gravity is the net force on such a satellite. Scientists have actually measured the mass of Earth this way. (Chapter 7)

$$F_{\text{net}} = \frac{m_s v^2}{r} = \frac{G m_s m_e}{r^2}$$

Since, $v = \frac{2\pi r}{T}$

$$\left(\frac{m_s}{r}\right) \left(\frac{4\pi^2 r^2}{T^2}\right) = \frac{G m_s m_e}{r^2}$$

$$m_e = \frac{4\pi^2 r^2}{G T^2}$$

$$= \frac{4\pi^2(1.0 \times 10^7 \text{ m})^3}{(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(9.9 \times 10^3 \text{ s})^2}$$

$$= 6.0 \times 10^{24} \text{ kg}$$

101. A 5.00-g bullet is fired with a velocity of 100.0 m/s toward a 10.00-kg stationary solid block resting on a frictionless surface. (Chapter 9)

**a.** What is the change in momentum of the bullet if it is embedded in the block?

$$m_b v_{b1} = m_b v_2 - m_w v_2$$

$$= (m_b + m_w)v_2$$
Chapter 11 continued

so \( v_2 = \frac{m_b v_{b1}}{m_b + m_w} \)

Then,

\[
\Delta p_v = m_b (v_2 - v_{b1}) \\
= m_b \left( \frac{m_b v_{b1}}{m_b + m_w} - v_{b1} \right) \\
= m_b v_{b1} \left( \frac{m_b}{m_b + m_w} - 1 \right) \\
= -m_b \frac{m_w}{m_b + m_w} v_{b1} \\
= -\frac{(5.00 \times 10^{-3} \text{ kg})(10.00 \text{ kg})}{5.00 \times 10^{-3} \text{ kg} + 10.00 \text{ kg}} (100.0 \text{ m/s}) \\
= -0.500 \text{ kg} \cdot \text{m/s}
\]

b. What is the change in momentum of the bullet if it ricochets in the opposite direction with a speed of 99 m/s?

\[
\Delta p_v = m_b (v_2 - v_{b1}) \\
= (5.00 \times 10^{-3} \text{ kg}) (-99.0 \text{ m/s} - 100.0 \text{ m/s}) \\
= -0.995 \text{ kg} \cdot \text{m/s}
\]

c. In which case does the block end up with a greater speed?

When the bullet ricochets, its change in momentum is larger in magnitude, and so is the block’s change in momentum, so the block ends up with a greater speed.

102. An automobile jack must exert a lifting force of at least 15 kN.

a. If you want to limit the effort force to 0.10 kN, what mechanical advantage is needed?

\[
MA = \frac{15 \text{ kN}}{0.10 \text{ kN}} = 150
\]

b. If the jack is 75% efficient, over what distance must the effort force be exerted in order to raise the auto 33 cm?

\[
IMA = \frac{MA}{e} = 2.0 \times 10^2.
\]

Since \( \frac{d_e}{d_r} = IMA \),

\[
d_e = \frac{IMA}{d_r} = (2.0 \times 10^2)(33 \text{ cm}) \\
= 66 \text{ m}
\]

Challenge Problem

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A bullet of mass \( m \), moving at speed \( v_1 \), goes through a motionless wooden block and exits with speed \( v_2 \). After the collision, the block, which has mass \( m_B \), is moving.

1. What is the final speed \( v_B \) of the block?

Conservation of momentum:

\[
mv_1 = mv_2 + m_B v_B \\
m_B v_B = m(v_1 - v_2) \\
v_B = \frac{m(v_1 - v_2)}{m_B}
\]

2. How much energy was lost to the bullet?

For the bullet alone:

\[
KE_1 = \frac{1}{2} mv_1^2 \\
KE_2 = \frac{1}{2} mv_2^2 \\
\Delta KE = \frac{1}{2} m(v_1^2 - v_2^2)
\]

3. How much energy was lost to friction inside the block?

Energy lost to friction = \( KE_1 - KE_2 - KE_{\text{block}} \)

\[
E_{\text{lost}} = \frac{1}{2} mv_1^2 - \frac{1}{2} mv_2^2 - \frac{1}{2} m_B v_B^2
\]