CHAPTER 19

Interference and Diffraction

Practice Problems

19.1 Interference pages 515–523

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1. Violet light falls on two slits separated by 1.90×10^{-5} m. A first-order bright band appears 13.2 mm from the central bright band on a screen 0.600 m from the slits. What is λ?

$$\lambda = \frac{xd}{L}$$

$$= \frac{(13.2\times10^{-3}\ \text{m})(1.90\times10^{-5}\ \text{m})}{0.600\ \text{m}}$$

$$= 418\ \text{nm}$$

2. Yellow-orange light from a sodium lamp of wavelength 596 nm is aimed at two slits that are separated by 1.90×10^{-5} m. What is the distance from the central band to the first-order yellow band if the screen is 0.600 m from the slits?

$$x = \frac{\lambda L}{d}$$

$$= \frac{(13.2\times10^{-3}\ \text{m})(1.90\times10^{-5}\ \text{m})}{0.600\ \text{m}}$$

$$= 1.88\times10^{-2}\ \text{m} = 18.8\ \text{mm}$$

3. In a double-slit experiment, physics students use a laser with $\lambda = 632.8$ nm. A student places the screen 1.000 m from the slits and finds the first-order bright band 65.5 mm from the central line. What is the slit separation?

$$\lambda = \frac{xd}{L}$$

$$d = \frac{\lambda L}{x}$$

$$= \frac{(632.8\times10^{-9}\ \text{m})(1.000\ \text{m})}{65.5\times10^{-3}\ \text{m}}$$

$$= 9.66\times10^{-6}\ \text{m} = 9.66\ \mu\text{m}$$

4. Yellow-orange light with a wavelength of 596 nm passes through two slits that are separated by 2.25×10^{-5} m and makes an interference pattern on a screen. If the distance from the central line to the first-order yellow band is 2.00×10^{-2} m, how far is the screen from the slits?

$$\lambda = \frac{xd}{L}$$

$$L = \frac{xd}{\lambda}$$

$$= \frac{(2.00\times10^{-2}\ \text{m})(2.25\times10^{-5}\ \text{m})}{596\times10^{-9}\ \text{m}}$$

$$= 0.755\ \text{m}$$

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5. In the situation in Example Problem 2, what would be the thinnest film that would create a reflected red ($\lambda = 635$ nm) band?

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_{\text{oil}}}$$

For the thinnest film, $m = 0$.

$$t = \left(\frac{1}{4}\right)\frac{\lambda}{n_{\text{oil}}}$$

$$= \frac{635\ \text{nm}}{(4)(1.45)}$$

$$= 109\ \text{nm}$$

6. A glass lens has a nonreflective coating placed on it. If a film of magnesium fluoride, $n = 1.38$, is placed on the glass, $n = 1.52$, how thick should the layer be to keep yellow-green light from being reflected?

Because $n_{\text{film}} > n_{\text{air}}$, there is a phase inversion on the first reflection.
Because $n_{\text{glass}} > n_{\text{film}}$, there is a phase inversion on the second reflection.
For destructive interference to keep yellow-green from being reflected:

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_{\text{film}}}$$

For the thinnest film, $m = 0$.

$$t = \left(\frac{1}{4}\right)\frac{\lambda}{n_{film}}$$

$$= \frac{555\text{ nm}}{(4)(1.38)}$$

$$= 101\text{ nm}$$

7. A silicon solar cell has a nonreflective coating placed on it. If a film of sodium monoxide, $n = 1.45$, is placed on the silicon, $n = 3.5$, how thick should the layer be to keep yellow-green light ($\lambda = 555$ nm) from being reflected?

Because $n_{film} > n_{air}$, there is a phase inversion on the first reflection. Because $n_{silicon} > n_{film}$, there is a phase inversion on the second reflection. For destructive interference to keep yellow-green from being reflected:

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_{film}}$$

For the thinnest film, $m = 0$.

$$t = \left(\frac{1}{4}\right)\frac{\lambda}{n_{film}}$$

$$= \frac{555\text{ nm}}{(4)(1.45)}$$

$$= 95.7\text{ nm}$$

8. You can observe thin-film interference by dipping a bubble wand into some bubble solution and holding the wand in the air. What is the thickness of the thinnest soap film at which you would see a black stripe if the light illuminating the film has a wavelength of 521 nm? Use $n = 1.33$.

Because $n_{film} > n_{air}$, there is a phase change on the first reflection. Because $n_{air} < n_{film}$, there is no phase change on the second reflection. For destructive interference to get a black stripe

$$2t = \frac{m\lambda}{n_{film}}$$

For the thinnest film, $m = 1$.

$$t = \frac{\lambda}{2n_{film}}$$

$$= \frac{521\text{ nm}}{(2)(1.33)}$$

$$= 196\text{ nm}$$

9. What is the thinnest soap film ($n = 1.33$) for which light of wavelength 521 nm will constructively interfere with itself?

For constructive interference

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_{film}}$$

For the thinnest film, $m = 0$.

$$t = \left(\frac{1}{4}\right)\frac{\lambda}{n_{film}}$$

$$= \frac{521\text{ nm}}{(4)(1.33)}$$

$$= 97.9\text{ nm}$$

Section Review

19.1 Interference pages 515–523

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10. **Film Thickness** Lucien is blowing bubbles and holds the bubble wand up so that a soap film is suspended vertically in the air. What is the second thinnest width of the soap film at which he could expect to see a bright stripe if the light illuminating the film has a wavelength of 575 nm? Assume the soap solution has an index of refraction of 1.33.

There is one phase inversion, so constructive interference will be when

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_{film}}$$

For the second thinnest thickness, $m = 1$.

$$t = \left(\frac{3}{4}\right)\frac{\lambda}{n_{film}}$$

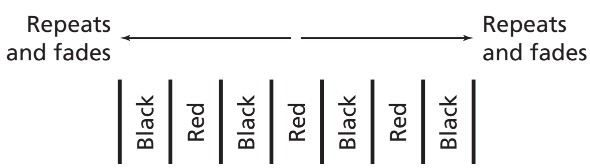
$$= \frac{(3)(575\text{ nm})}{(4)(1.33)}$$

$$= 324\text{ nm}$$

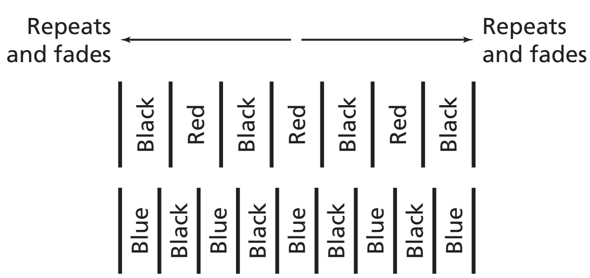
11. **Bright and Dark Patterns** Two very narrow slits are cut close to each other in a large piece of cardboard. They are illuminated by monochromatic red light. A sheet of white paper is placed far from the slits, and a pattern of bright and dark bands is seen on the paper. Describe how a wave behaves when it encounters a slit, and explain why some regions are bright while others are dark.

    When a wave encounters a slit, the wave bends. Light is diffracted by the slits. Light from one slit interferes with light from the other. If interference is constructive, there is a bright band; if destructive, the region is dark.

12. **Interference Patterns** Sketch the pattern described in problem 11.



13. **Interference Patterns** Sketch what happens to the pattern in problem 11 when the red light is replaced by blue light.



    The light bands become more closely spaced.

14. **Film Thickness** A plastic reflecting film ($n = 1.83$) is placed on an auto glass window ($n = 1.52$).

    a. What is the thinnest film that will reflect yellow-green light?

    Because $n_{film} > n_{air}$, there is a phase change on the first reflection. Because $n_{glass} < n_{film}$, there is not a phase change on the second reflection. For constructive interference to reflect yellow-green light:

    $$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_{film}}$$

    For the thinnest film, $m = 0$.

    $$t = \left(\frac{1}{4}\right)\frac{\lambda_{color}}{n_{film}} = \frac{555\text{ nm}}{(4)(1.83)} = 75.8\text{ nm}$$

    b. Unfortunately, a film this thin cannot be manufactured. What is the next-thinnest film that will produce the same effect?

    For the next thinnest film, $m = 1$.

    $$t = \left(\frac{3}{4}\right)\frac{\lambda_{color}}{n_{film}} = \frac{(3)(555\text{ nm})}{(4)(1.83)} = 227\text{ nm}$$

15. **Critical Thinking** The equation for wavelength from a double-slit experiment uses the simplification that θ is small so that $\sin\theta \approx \tan\theta$. Up to what angle is this a good approximation when your data has two significant figures? Would the maximum angle for a valid approximation increase or decrease as you increase the precision of your angle measurement?

    $\sin\theta = \tan\theta$ to two significant digits up to 9.9°. An increase in the precision of the measurement reduces this angle to 2.99°.

Practice Problems

19.2 Diffraction pages 524–531

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16. Monochromatic green light of wavelength 546 nm falls on a single slit with a width of 0.095 mm. The slit is located 75 cm from a screen. How wide will the central bright band be?

$$\lambda = \frac{x_{min}w}{L}$$

$$x_{min} = \frac{\lambda L}{w}$$

$$= \frac{(5.46\times10^{-7}\text{ m})(0.75\text{ m})}{9.5\times10^{-5}\text{ m}}$$

$$= 4.3\text{ mm}$$

17. Yellow light with a wavelength of 589 nm passes through a slit of width 0.110 mm and makes a pattern on a screen. If the width of the central bright band is 2.60×10^{-2} m, how far is it from the slits to the screen?

$$2x_1 = \frac{2\lambda L}{w}$$

$$L = \frac{(2x_1)w}{2\lambda}$$

$$= \frac{(2.60\times10^{-2}\text{ m})(0.110\times10^{-3}\text{ m})}{(2)(589\times10^{-9}\text{ m})}$$

$$= 2.43\text{ m}$$

18. Light from a He-Ne laser (λ = 632.8 nm) falls on a slit of unknown width. A pattern is formed on a screen 1.15 m away, on which the central bright band is 15 mm wide. How wide is the slit?

$$2x_1 = \frac{2\lambda L}{w}$$

$$w = \frac{2\lambda L}{2x_1}$$

$$= \frac{(2)(632.8\times10^{-9}\text{ m})(1.15\text{ m})}{15\times10^{-3}\text{ m}}$$

$$= 9.7\times10^{-5}\text{ m}$$

$$= 97\ \mu\text{m}$$

19. Yellow light falls on a single slit 0.0295 mm wide. On a screen that is 60.0 cm away, the central bright band is 24.0 mm wide. What is the wavelength of the light?

$$2x_1 = \frac{2\lambda L}{w}$$

$$\lambda = \frac{(2x_1)w}{2L}$$

$$= \frac{(24.0\times10^{-3}\text{ m})(0.0295\times10^{-3}\text{ m})}{(2)(60.0\times10^{-2}\text{ m})}$$

$$= 5.90\times10^{2}\text{ nm}$$

20. White light falls on a single slit that is 0.050 mm wide. A screen is placed 1.00 m away. A student first puts a blue-violet filter (λ = 441 nm) over the slit, then a red filter (λ = 622 nm). The student measures the width of the central bright band.

a. Which filter produced the wider band?

Red, because central peak width is proportional to wavelength.

b. Calculate the width of the central bright band for each of the two filters.

$$2x_1 = \frac{2\lambda L}{w}$$

For blue,

$$2x_1 = \frac{2(4.41\times10^{-7}\text{ m})(1.00\text{ m})}{5.0\times10^{-5}\text{ m}}$$

$$= 18\text{ mm}$$

For red,

$$2x_1 = \frac{2(6.22\times10^{-7}\text{ m})(1.00\text{ m})}{5.0\times10^{-5}\text{ m}}$$

$$= 25\text{ mm}$$

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21. White light shines through a grating onto a screen. Describe the pattern that is produced.

A full spectrum of color is seen. Because of the variety of wavelengths, dark fringes of one wavelength would be filled by bright fringes of another color.

22. If blue light of wavelength 434 nm shines on a diffraction grating and the spacing of the resulting lines on a screen that is 1.05 m away is 0.55 m, what is the spacing between the slits in the grating?

$$\lambda = d\sin\theta$$

$$d = \frac{\lambda}{\sin\theta}\text{ where }\theta = \tan^{-1}\left(\frac{x}{L}\right)$$

$$= \frac{\lambda}{\sin\left(\tan^{-1}\left(\frac{x}{L}\right)\right)}$$

$$= \frac{434\times10^{-9}}{\sin\left(\tan^{-1}\left(\frac{0.55\text{ m}}{1.05\text{ m}}\right)\right)}$$

$$= 9.4\times10^{-7}\text{ m}$$

23. A diffraction grating with slits separated by 8.60×10^{-7} m is illuminated by violet light with a wavelength of 421 nm. If the screen is 80.0 cm from the grating, what is the separation of the lines in the diffraction pattern?

$$\lambda = d \sin \theta$$

$$\sin \theta = \frac{\lambda}{d}$$

$$\tan \theta = \frac{x}{L}$$

$$x = L \tan \theta$$

$$= L \tan \left(\sin^{-1}\left(\frac{\lambda}{d}\right)\right)$$

$$= (0.800\text{ m})\left(\tan\left(\sin^{-1}\left(\frac{421\times10^{-9}\text{ m}}{8.60\times10^{-7}\text{ m}}\right)\right)\right)$$

$$= 0.449\text{ m}$$

24. Blue light shines on the DVD in Example Problem 3. If the dots produced on a wall that is 0.65 m away are separated by 58.0 cm, what is the wavelength of the light?

$$\lambda = d \sin \theta = d \sin\left(\tan^{-1}\left(\frac{x}{L}\right)\right)$$

$$= (7.41\times10^{-7}\text{ m})\left(\sin\left(\tan^{-1}\left(\frac{0.58\text{ m}}{0.65\text{ m}}\right)\right)\right)$$

$$= 490\text{ nm}$$

25. Light of wavelength 632 nm passes through a diffraction grating and creates a pattern on a screen that is 0.55 m away. If the first bright band is 5.6 cm from the central bright band, how many slits per centimeter does the grating have?

$$\lambda = d \sin \theta$$

There is one slit per distance *d*, so $\frac{1}{d}$ gives slits per centimeter.

$$d = \frac{\lambda}{\sin \theta} = \frac{\lambda}{\sin\left(\tan^{-1}\left(\frac{x}{L}\right)\right)}$$

$$= \frac{632\times10^{-9}\text{ m}}{\sin\left(\tan^{-1}\left(\frac{0.056\text{ m}}{0.55\text{ m}}\right)\right)}$$

$$= 6.2\times10^{-6}\text{ m} = 6.2\times10^{-4}\text{ cm}$$

$$\frac{1\text{ slit}}{6.2\times10^{-4}\text{ cm}} = 1.6\times10^{3}\text{ slits/cm}$$

Section Review

19.2 Diffraction pages 524–531

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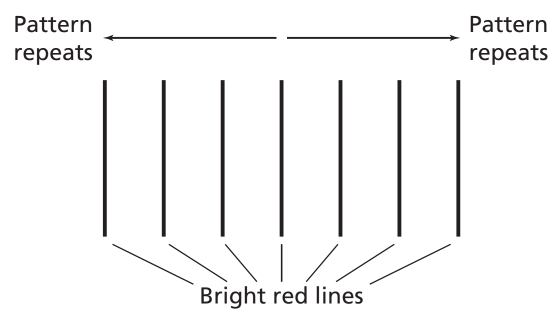
26. Distance Between First-Order Dark Bands Monochromatic green light of wavelength 546 nm falls on a single slit of width 0.080 mm. The slit is located 68.0 cm from a screen. What is the separation of the first dark bands on each side of the central bright band?

$$2x_{min} = \frac{2\lambda L}{w}$$

$$= \frac{(2)(546\times10^{-9}\text{ m})(68.0\times10^{-2}\text{ m})}{0.080\times10^{-3}\text{ m}}$$

$$= 9.3\text{ mm}$$

27. Diffraction Patterns Many narrow slits are close to each other and equally spaced in a large piece of cardboard. They are illuminated by monochromatic red light. A sheet of white paper is placed far from the slits, and a pattern of bright and dark bands is visible on the paper. Sketch the pattern that would be seen on the screen.



Band spacing is exactly the same as in the pattern produced by the two slits, but now light bands are much thinner and separated by wider dark bands.

28. Line Spacing You shine a red laser light through one diffraction grating and form a pattern of red dots on a screen. Then you substitute a second diffraction grating for the first one, forming a different pattern. The dots produced by the first grating are spread out more than those produced by the second. Which grating has more lines per millimeter?

$\lambda \approx \frac{xd}{L}$, so the greater the dot spacings, x, the narrower the slit spacing, d, and thus more lines per millimeter.

29. **Rayleigh Criterion** The brightest star in the winter sky in the northern hemisphere is Sirius. In reality, Sirius is a system of two stars that orbit each other. If the *Hubble Space Telescope* (diameter 2.4 m) is pointed at the Sirius system, which is 8.44 light-years from Earth, what is the minimum separation there would need to be between the stars in order for the telescope to be able to resolve them? Assume that the average light coming from the stars has a wavelength of 550 nm.

$$x_{obj} = \frac{1.22\lambda L_{obj}}{D}$$

$$= \frac{1.22(330\times10^{-9}\text{ m})(7.99\times10^{16}\text{ m})}{2.4\text{ m}}$$

$$= 2.2\times10^{10}\text{ m}$$

30. **Critical Thinking** You are shown a spectrometer, but do not know whether it produces its spectrum with a prism or a grating. By looking at a white-light spectrum, how could you tell?

Determine if the violet or the red end of the spectrum makes the largest angle with the direction of the beam of incident white light. A prism bends the violet end of the spectrum the most, whereas a grating diffracts red wavelengths the most.

Chapter Assessment

Concept Mapping

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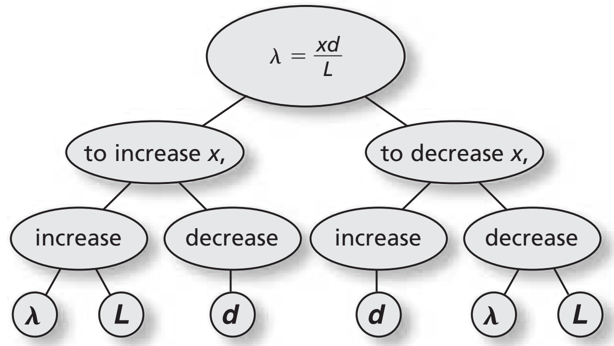
31. Monochromatic light of wavelength λ illuminates two slits in a Young's double-slit experiment setup that are separated by a distance, d. A pattern is projected onto a screen a distance, L, away from the slits. Complete the following concept map using λ, L, and d to indicate how you could vary them to produce the indicated change in the spacing between adjacent bright bands, x.



Mastering Concepts

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32. Why is it important that monochromatic light was used to make the interference pattern in Young's interference experiment? (19.1)

When monochromatic light is used, you get a sharp interference pattern; if you use white light, you get sets of colored bands.

33. Explain why the position of the central bright band of a double-slit interference pattern cannot be used to determine the wavelength of the light waves. (19.1)

All wavelengths produce the line in the same place.

34. Describe how you could use light of a known wave-length to find the distance between two slits. (19.1)

Let the light fall on the double slit, and let the interference pattern fall on a sheet of paper. Measure the spacings between the bright bands, x, and use the equation $d = \frac{\lambda L}{x}$.

35. Describe in your own words what happens in thin-film interference when a dark band is produced by light shining on a soap film suspended in air. Make sure you include in your explanation how the wavelength of the light and thickness of the film are related. (19.1)

When the light strikes the front of the film, some reflects off this surface and some passes through the film and reflects off the back surface of the film. When light reflects off a medium with a higher index of refraction, it undergoes a phase shift of one-half wavelength; this happens to the light that initially reflects. In order for a dark band to be produced, the two light rays must be one-half wavelength out of phase. If the thickness of the film is such that the ray reflecting off the back surface goes through a whole number of cycles while passing through the film, the light rays arriving at your eye will be out of phase and destructively interfere. Remember that the wavelength is altered by the index of refraction of the film, so that the thickness of the film must equal a multiple of half a wavelength of the light, divided by the film's index of refraction.

36. White light shines through a diffraction grating. Are the resulting red lines spaced more closely or farther apart than the resulting violet lines? Why? (19.2)

The spacing is directly proportional to the wavelength, and because red light has a longer wavelength than violet, the red lines will be spaced farther apart than the violet lines.

37. Why do diffraction gratings have large numbers of slits? Why are these slits so close together? (19.2)

The large number of grooves in diffraction gratings increases the intensity of the diffraction patterns. The grooves are close together, producing sharper images of light.

38. Why would a telescope with a small diameter not be able to resolve the images of two closely spaced stars? (19.2)

Small apertures have large diffraction patterns that limit resolution.

39. For a given diffraction grating, which color of visible light produces a bright line closest to the central bright band? (19.2)

violet light, the color with the smallest wavelength

Applying Concepts

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40. For each of the following examples, indicate whether the color is produced by thin-film interference, refraction, or the presence of pigments.

a. soap bubbles

interference

b. rose petals

pigments

c. oil films

interference

d. a rainbow

refraction

41. How can you tell whether a pattern is produced by a single slit or a double slit?

A double-slit interference pattern consists of equally spaced lines of almost equal brightness. A single-slit diffraction pattern has a bright, broad central band and dimmer side bands.

42. Describe the changes in a single-slit diffraction pattern as the width of the slit is decreased.

The bands get wider and dimmer.

43. **Science Fair** At a science fair, one exhibition is a very large soap film that has a fairly consistent width. It is illuminated by a light with a wavelength of 432 nm, and nearly the entire surface appears to be a lovely shade of purple. What would you see in the following situations?

a. the film thickness was doubled

complete destructive interference

b. the film thickness was increased by half a wavelength of the illuminating light

complete constructive interference

c. the film thickness was decreased by one quarter of a wavelength of the illuminating light

complete destructive interference

44. What are the differences in the characteristics of the diffraction patterns formed by diffraction gratings containing 10^4 lines/cm and 10^5 lines/cm?

The lines in the diffraction pattern are narrower for the 10^5 lines/cm grating.

45. Laser Pointer Challenge You have two laser pointers, a red one and a green one. Your friends Mark and Carlos disagree about which has the longer wavelength. Mark insists that red light has a longer wavelength, while Carlos is sure that green has the longer wavelength. You have a diffraction grating handy. Describe what demonstration you would do with this equipment and how you would explain the results to Carlos and Mark to settle their disagreement.

Shine each laser pointer through the grating onto a nearby wall. The color with the longer wavelength will produce dots with a greater spacing on the wall because the spacing is directly proportional to the wavelength. (Mark is correct; red light has a longer wavelength than green light.)

46. Optical Microscope Why is blue light used for illumination in an optical microscope?

Less diffraction results from the short wavelength of blue light.

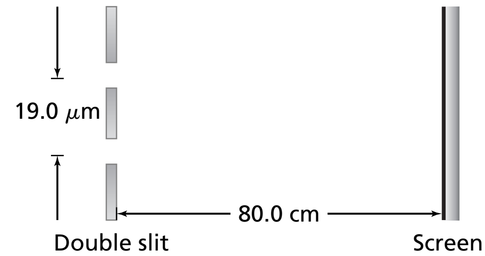
Mastering Problems

19.1 Interference

pages 536–537

Level 1

47. Light falls on a pair of slits 19.0 μm apart and 80.0 cm from a screen, as shown in **Figure 19-17.** The first-order bright band is 1.90 cm from the central bright band. What is the wavelength of the light?



■ **Figure 19-17** (Not to scale)

$$\lambda = \frac{xd}{L}$$

$$= \frac{(19.0\times10^{-6}\text{ m})(1.90\times10^{-2}\text{ m})}{80.0\times10^{-2}\text{ m}}$$

$$= 451\text{ nm}$$

48. Oil Slick After a short spring shower, Tom and Ann take their dog for a walk and notice a thin film of oil ($n = 1.45$) on a puddle of water, producing different colors. What is the minimum thickness of a place where the oil creates constructive interference for light with a wavelength of 545 nm?

There is one phase inversion, so constructive interference will be when

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_{\text{film}}}$$

For the minimum thickness, $m = 0$.

$$t = \left(\frac{1}{4}\right)\frac{\lambda}{n_{\text{film}}}$$

$$= \frac{545\text{ nm}}{(4)(1.45)}$$

$$= 94.0\text{ nm}$$

Level 2

49. Light of wavelength 542 nm falls on a double slit. First-order bright bands appear 4.00 cm from the central bright band. The screen is 1.20 m from the slits. How far apart are the slits?

$$\lambda = \frac{xd}{L}$$

$$d = \frac{\lambda L}{x}$$

$$= \frac{(5.42\times10^{-7}\text{ m})(1.20\text{ m})}{4.00\times10^{-2}\text{ m}}$$

$$= 16.3\ \mu\text{m}$$

50. Insulation Film Winter is approaching and Alejandro is helping to cover the windows in his home with thin sheets of clear plastic (n = 1.81) to keep the drafts out. After the plastic is taped up around the windows such that there is air between the plastic and the glass panes, the plastic is heated with a hair dryer to shrink-wrap the window. The thickness of the plastic is altered during this process. Alejandro notices a place on the plastic where there is a blue stripe of color. He realizes that this is created by thin-film interference. What are three possible thicknesses of the portion of the plastic where the blue stripe is produced if the wavelength of the light is 4.40×10^2 nm?

There is one phase inversion, so constructive interference will be when

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_{film}}$$

Three possible thicknesses occur at m = 0, 1, and 2.

$$t = \left(\frac{1}{4}\right)\frac{\lambda}{n_{film}} \text{ for } m = 0$$

$$= \frac{4.40\times10^2 \text{ nm}}{(4)(1.81)}$$

$$= 60.8 \text{ nm}$$

$$t = \left(\frac{3}{4}\right)\frac{\lambda}{n_{film}} \text{ for } m = 1$$

$$= \frac{(3)(4.40\times10^2 \text{ nm})}{(4)(1.81)}$$

$$= 182 \text{ nm}$$

$$t = \left(\frac{5}{4}\right)\frac{\lambda}{n_{film}} \text{ for } m = 2$$

$$= \frac{(5)(4.40\times10^2 \text{ nm})}{(4)(1.81)}$$

$$= 304 \text{ nm}$$

Level 3

51. Samir shines a red laser pointer through three different double-slit setups. In setup A, the slits are separated by 0.150 mm and the screen is 0.60 m away from the slits. In setup B, the slits are separated by 0.175 mm and the screen is 0.80 m away. Setup C has the slits separated by 0.150 mm and the screen a distance of 0.80 m away. Rank the three setups according to the separation between the central bright band and the first-order bright band, from least to most separation. Specifically indicate any ties.

$$\lambda = \frac{xd}{L}$$

$$x = \frac{\lambda L}{d}$$

Because λ is the same for each setup, calculate $\frac{x}{\lambda}$ to compare the setups.

$$\frac{x}{\lambda} = \frac{L}{d}$$

Setup A:

$$= \frac{0.60 \text{ m}}{1.50\times10^{-4} \text{ m}}$$

$$= 4.0\times10^3$$

Setup B:

$$= \frac{0.80 \text{ m}}{1.75\times10^{-4} \text{ m}}$$

$$= 4.6\times10^3$$

Setup C:

$$= \frac{0.80 \text{ m}}{1.50\times10^{-4} \text{ m}}$$

$$= 5.3\times10^3$$

$$x_C > x_B > x_A$$

19.2 Diffraction

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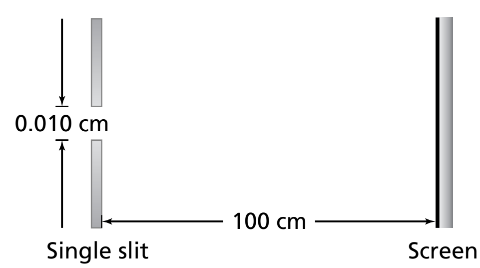
Level 1

52. Monochromatic light passes through a single slit with a width of 0.010 cm and falls on a screen 100 cm away, as shown in **Figure 19-18.** If the width of the central band is 1.20 cm, what is the wavelength of the light?



■ **Figure 19-18** (Not to scale)

$$2x_1 = \frac{2\lambda L}{w}$$

$$\lambda = \frac{xw}{L}$$

$$= \frac{(0.60\ \text{cm})(0.010\ \text{cm})}{100\ \text{cm}}$$

$$= 600\ \text{nm}$$

53. A good diffraction grating has 2.5×10^3 lines per cm. What is the distance between two lines in the grating?

$$d = \frac{1}{2.5\times10^3\ \text{lines/cm}}$$

$$= 4.0\times10^{-4}\ \text{cm}$$

Level 2

54. Light with a wavelength of 4.5×10^{-5} cm passes through a single slit and falls on a screen 100 cm away. If the slit is 0.015 cm wide, what is the distance from the center of the pattern to the first dark band?

$$2x_1 = \frac{2\lambda L}{w}$$

$2x_1$ is the width of the bright band, so to get the distance from the center to the first dark band, divide by 2.

$$x_1 = \frac{\lambda L}{w}$$

$$= \frac{(4.5\times10^{-5}\ \text{cm})(100\ \text{m})}{0.015\ \text{cm}}$$

$$= 0.3\ \text{cm}$$

55. ***Hubble Space Telescope*** Suppose the *Hubble Space Telescope,* 2.4 m in diameter, is in orbit 1.0×10^5 m above Earth and is turned to view Earth, as shown in **Figure 19-19.** If you ignore the effect of the atmosphere, how large an object can the telescope resolve? Use $\lambda = 5.1\times10^{-7}$ m.

■ **Figure 19-19**

$$\frac{x_{obj}}{L_{obj}} = \frac{1.22\lambda}{D}$$

$$x_{obj} = \frac{1.22\lambda L_{obj}}{D}$$

$$= \frac{(1.22)(5.1\times10^{-7}\ \text{m})(1.0\times10^5\ \text{m})}{2.4\ \text{m}}$$

$$= 2.6\times10^{-2}\ \text{m}$$

$$= 2.6\ \text{cm}$$

Level 3

56. Monochromatic light with a wavelength of 425 nm passes through a single slit and falls on a screen 75 cm away. If the central bright band is 0.60 cm wide, what is the width of the slit?

$$2x_1 = \frac{2\lambda L}{w}$$

$$w = \frac{2\lambda L}{2x_1} = \frac{\lambda L}{x_1}$$

$$x_1 = \left(\frac{1}{2}\right)(2x_1) = 0.30\ \text{cm}$$

$$= \frac{(4.25\times10^{-5}\ \text{cm})(75\ \text{cm})}{0.30\ \text{cm}}$$

$$= 1.1\times10^{-2}\ \text{cm}$$

57. **Kaleidoscope** Jennifer is playing with a kaleidoscope from which the mirrors have been removed. The eyehole at the end is 7.0 mm in diameter. If she can just distinguish two bluish-purple specks on the other end of the kaleidoscope separated by 40 μm, what is the length of the kaleidoscope? Use $\lambda = 650$ nm and assume that the resolution is diffraction limited through the eyehole.

$$x_{obj} = \frac{1.22\lambda L_{obj}}{D}$$

$$L_{obj} = \frac{x_{obj}D}{1.22\lambda}$$

$$= \frac{(40\times10^{-6}\ \text{m})(7.0\times10^{-3}\ \text{m})}{(1.22)(650\times10^{-9}\ \text{m})}$$

$$= 0.4\ \text{m}$$

58. Spectroscope A spectroscope uses a grating with 12,000 lines/cm. Find the angles at which red light, 632 nm, and blue light, 421 nm, have first-order bright lines.

$$d = \frac{1}{12{,}000 \text{ lines/cm}} = 8.33\times10^{-5} \text{ cm}$$

$$\lambda = d \sin \theta$$

$$\sin \theta = \frac{\lambda}{d}$$

For red light,

$$\theta = \sin^{-1}\left(\frac{6.32\times10^{-5} \text{ cm}}{8.33\times10^{-5} \text{ cm}}\right)$$

$$= 49.3°$$

For blue light,

$$\theta = \sin^{-1}\left(\frac{4.21\times10^{-5} \text{ cm}}{8.33\times10^{-5} \text{ cm}}\right)$$

$$= 30.3°$$

Mixed Review

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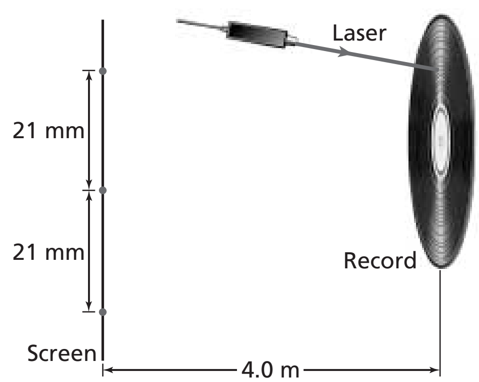
Level 1

59. Record Marie uses an old $33\frac{1}{3}$ rpm record as a diffraction grating. She shines a laser, $\lambda = 632.8$ nm, on the record, as shown in **Figure 19-20.** On a screen 4.0 m from the record, a series of red dots 21 mm apart are visible.



■ **Figure 19-20** (Not to scale)

a. How many ridges are there in a centimeter along the radius of the record?

$$\lambda = d \sin \theta$$

$$d = \frac{\lambda}{\sin \theta} = \frac{\lambda}{\sin\left(\tan^{-1}\left(\frac{x}{L}\right)\right)}$$

$$= \frac{632.8\times10^{-9} \text{ m}}{\sin\left(\tan^{-1}\left(\frac{21\times10^{3} \text{ m}}{4.0 \text{ m}}\right)\right)}$$

$$= 1.2\times10^{-4} \text{ m} = 1.2\times10^{-2} \text{ cm}$$

$$\frac{1}{d} = \frac{1}{1.2\times10^{-2} \text{ cm}} = 83 \text{ ridges/cm}$$

b. Marie checks her results by noting that the ridges represent a song that lasts 4.01 minutes and takes up 16 mm on the record. How many ridges should there be in a centimeter?

Number of ridges is

(4.01 min)(33.3 rev/min) = 134 ridges

$$\frac{134 \text{ ridges}}{1.6 \text{ cm}} = 84 \text{ ridges/cm}$$

Level 2

60. An anti-reflective coating, $n = 1.2$, is applied to a lens. If the thickness of the coating is 125 nm, what is (are) the color(s) of light for which complete destructive interference will occur? *Hint: Assume the lens is made out of glass.*

Because $n_{film} > n_{air}$, there is a phase inversion on the first reflection. Because $n_{lens} = 1.52 > n_{film}$, there is a phase inversion on the second reflection.

For destructive interference:

$$2d = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_{film}}$$

$$\lambda = \frac{2dn_{film}}{\left(m + \frac{1}{2}\right)}$$

$$= \frac{(2)(125 \text{ nm})(1.2)}{\left(m + \frac{1}{2}\right)}$$

$$= \left(m + \frac{1}{2}\right)^{-1}(3.0\times10^{2} \text{ nm})$$

For $m = 0$

$$= \left(\frac{1}{2}\right)^{-1}(3.0\times10^{2} \text{ nm})$$

$$= 6.0\times10^{2} \text{ nm}$$

The light is reddish-orange. For other values of m, the wavelength is shorter than that of light.

Level 3

61. Camera When a camera with a 50-mm lens is set at f/8, its aperture has an opening 6.25 mm in diameter.

a. For light with $\lambda = 550$ nm, what is the resolution of the lens? The film is 50.0 mm from the lens.

$$x_{obj} = \frac{1.22\lambda L_{obj}}{D}$$

$$= \frac{(1.22)(5.5\times10^{-4}\ \text{mm})(50.0\ \text{mm})}{6.25\ \text{mm}}$$

$$= 5.4\times10^{-3}\ \text{mm}$$

b. The owner of a camera needs to decide which film to buy for it. The expensive one, called fine-grained film, has 200 grains/mm. The less costly, coarse-grained film has only 50 grains/mm. If the owner wants a grain to be no smaller than the width of the central bright spot calculated in part **a,** which film should he purchase?

Central bright band width

$$2x' = 10.7\times10^{-3}\ \text{mm}$$

The 200 grains/mm film has $\frac{1}{200\ \text{mm}}$ **between grains** $= 5\times10^{-3}$ **mm, so this film will work.**

The 50 grains/mm has $\frac{1}{50\ \text{mm}}$ **between grains** $= 20\times10^{-3}$ **mm, so this film won't work.**

Thinking Critically

page 538

62. Apply Concepts Yellow light falls on a diffraction grating. On a screen behind the grating, you see three spots: one at zero degrees, where there is no diffraction, and one each at +30° and −30°. You now add a blue light of equal intensity that is in the same direction as the yellow light. What pattern of spots will you now see on the screen?

A green spot at 0°, yellow spots at +30° and −30°, and two blue spots slightly closer in.

63. Apply Concepts Blue light of wavelength λ passes through a single slit of width w. A diffraction pattern appears on a screen. If you now replace the blue light with a green light of wavelength 1.5λ, to what width should you change the slit to get the original pattern back?

The angle of diffraction depends on the ratio of slit width to wavelength. Thus, you would increase the width to 1.5*w*.

64. Analyze and Conclude At night, the pupil of a human eye has an aperture diameter of 8.0 mm. The diameter is smaller in daylight. An automobile's headlights are separated by 1.8 m.

a. Based upon Rayleigh's criterion, how far away can the human eye distinguish the two headlights at night? *Hint: Assume a wavelength of 525 nm.*

$$x_{obj} = \frac{1.22\lambda L_{obj}}{D}$$

$$L_{obj} = \frac{x_{obj}D}{1.22\lambda}$$

$$= \frac{(8.0\times10^{-3}\ \text{m})(1.80\ \text{m})}{(1.22)(5.25\times10^{-7}\ \text{m})}$$

$$= 2.2\times10^{4}\ \text{m} = 22\ \text{km}$$

b. Can you actually see a car's headlights at the distance calculated in part **a?** Does diffraction limit your eyes' sensing ability? Hypothesize as to what might be the limiting factors.

No; a few hundred meters, not several kilometers, is the limit. Diffraction doesn't limit the sensing ability of your eyes. More probable factors are the refractive effects of the atmosphere, like those that cause stars to twinkle, or the limitations of the retina and the optic area of the brain to separate two dim sources.

Writing in Physics

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65. Research and describe Thomas Young's contributions to physics. Evaluate the impact of his research on the scientific thought about the nature of light.

Student answers will vary. Answers should include Young's two-slit experiment that allowed him to precisely measure the wavelength of light.

66. Research and interpret the role of diffraction in medicine and astronomy. Describe at least two applications in each field.

Student answers will vary. Answers could include diffraction in telescopes and microscopes, as well as spectroscopy.

Cumulative Review

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67. How much work must be done to push a 0.5-m^3 block of wood to the bottom of a 4-m-deep swimming pool? The density of wood is 500 kg/m^3. (Chapter 13)

The block would float, but to submerge it would require an extra force downward.

$W = Fd$

$F = F_{buoyancy} - F_g$

$F_g = \rho V g$

$= (500\ kg/m^3)(0.5\ m^3)(9.80\ m/s^2)$

$= 2450\ N$

$F_{buoyancy} = \rho V g$

$= (1000\ kg/m^3)(0.5\ m^3)(9.80\ m/s^2)$

$= 4900\ N$

$\text{Work} = (4900\ N - 2450\ N)(4\ m)$

$= 10\ kJ$

68. What are the wavelengths of microwaves in an oven if their frequency is 2.4 GHz? (Chapter 14)

$c = f\lambda$

$$\lambda = \frac{c}{f} = \frac{3.00\times10^8\ m/s}{2.4\times10^9\ Hz}$$

$= 0.12\ m$

69. Sound wave crests that are emitted by an airplane are 1.00 m apart in front of the plane, and 2.00 m apart behind the plane. (Chapter 15)

a. What is the wavelength of the sound in still air?

1.50 m

b. If the speed of sound is 330 m/s, what is the frequency of the source?

$v = f\lambda$

$$f = \frac{v}{\lambda} = \frac{330\ m/s}{1.50\ m} = 220\ Hz$$

c. What is the speed of the airplane?

The plane moves forward 0.50 m for every 1.50 m that the sound wave travels, so the plane's speed is one-third the speed of sound, or 110 m/s.

70. A concave mirror has a 48.0-cm radius. A 2.0-cm-tall object is placed 12.0 cm from the mirror. Calculate the image position and image height. (Chapter 17)

$$f = \frac{r}{2}$$

$$= \frac{48.0\ cm}{2}$$

$= 24.0\ cm$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$d_i = \frac{d_o f}{d_o - f}$$

$$= \frac{(12.0\ cm)(24.0\ cm)}{12.0\ cm - 24.0\ cm}$$

$= -24.0\ cm$

$$m \equiv \frac{h_i}{h_o} = \frac{-d_i}{d_o}$$

$$h_i = \frac{-d_i h_o}{d_o}$$

$$= \frac{-(-24.0\ cm)(2.0\ cm)}{12.0\ cm}$$

$= 4.0\ cm$

71. The focal length of a convex lens is 21.0 cm. A 2.00-cm-tall candle is located 7.50 cm from the lens. Use the thin-lens equation to calculate the image position and image height. (Chapter 18)

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$d_i = \frac{d_o f}{d_o - f}$$

$$= \frac{(7.50 \text{ cm})(21.0 \text{ cm})}{7.50 \text{ cm} - 21.0 \text{ cm}}$$

$$= -11.7 \text{ cm}$$

$$m \equiv \frac{h_i}{h_o} = \frac{-d_i}{d_o}$$

$$h_i = \frac{-d_i h_o}{d_o}$$

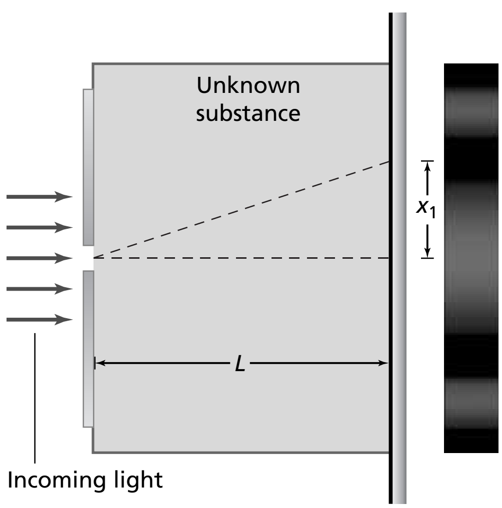
$$= \frac{-(-11.7 \text{ cm})(2.00 \text{ cm})}{7.50 \text{ cm}}$$

$$= 3.11 \text{ cm}$$

Challenge Problem

page 526

You have several unknown substances and wish to use a single-slit diffraction apparatus to determine what each one is. You decide to place a sample of an unknown substance in the region between the slit and the screen and use the data that you obtain to determine the identity of each substance by calculating its index of refraction.



1. Come up with a general formula for the index of refraction of an unknown substance in terms of the wavelength of the light, λ_{vacuum}, the width of the slit, w, the distance from the slit to the screen, L, and the distance between the central bright band and the first dark band, x_1.

Use (1) $\lambda = \frac{x_{min} w}{L}$, (2) $v_{substance} = \lambda f$, and (3) $n_{substance} = \frac{c}{v}$.

Combine (2) and (3).

$$n_{substance} = \frac{\lambda_{vacuum} f}{\lambda_{substance} f} = \frac{\lambda_{vacuum}}{\lambda_{substance}} \quad (4),$$

because the frequency remains constant as the light crosses a boundary.

Rewrite (1) in terms of a substance in the space between the slits and the screen.

$$\lambda_{substance} = \frac{x_{min} w}{L} \quad (5)$$

Combine (4) and (5) and solve for *x*.

$$n_{substance} = \frac{\lambda_{vacuum}}{\frac{x_{min} w}{L}}$$

$$x_{min} = \frac{\lambda_{vacuum} L}{n_{substance} w}$$

2. If the source you used had a wavelength of 634 nm, the slit width was 0.10 mm, the distance from the slit to the screen was 1.15 m, and you immersed the apparatus in water ($n_{substance}$ = 1.33), then what would you expect the width of the center band to be?

$$x = \frac{\lambda_{vacuum} L}{n_{substance} d}$$

$$= \frac{(634\times10^{-9} \text{ m})(1.15 \text{ m})}{(1.33)(0.10\times10^{-3} \text{ m})}$$

$$= 5.5\times10^{-3} \text{ m}$$